# Why Psychologists Should Always Report the *W*-test Instead of the *F*-Test ANOVA.

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The Supplemental Material, including the full R code for the simulations and plots can be obtained from <https://github.com/mdelacre/Welch-ANOVA>.

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# Abstract

When comparing independent groups, researchers in psychology commonly use Analysis of Variance (ANOVA), which is based on assumptions of normality and of homogeneity of variances. When these assumptions are not met, the classical ANOVA (*F-*test) can be severely biased, and leads to invalid statistical inferences. However, researchers rarely report whether test assumptions are met. We discuss why these assumptions will often not hold in psychological research and explain that this is problematic, especially for the assumption of homogeneity of variances. We show that the Welch’s ANOVA (*W*-test) controls the Type 1 error rate better than the *F*-test when the assumption of homogeneity of variance is not met, and loses little robustness compared to the *F-*test when the assumptions are met. Because assumption tests for the equality of variances often fail to provide an informative answer, we argue that the *W*-test should be the default choice in psychology.

Parametric tests are commonly used in psychological science to statistically test differences between means (Keselman et al., 1998). When there are more than two groups to compare, different tests that can be used, such as classical Analysis of Variance (*F-*test), Welch’s *W* ANOVA (*W-test*), the Alexander-Govern test, James’ test and the Brown-Forsythe ANOVA (*F\**-test). These tests differ in their underlying assumptions about whether data are sampled from a normal distribution or not, and whether these distributions have equal variances or not (Lix, Keselman, & Keselman, 1996). In psychological research, *F-*test is the default method to compare different groups (Erceg-Hurn & Mirosevich, 2008). The available alternative tests are reported considerably less often in the literature. Even though many tests psychologists report assume equal variances, researchers rarely provide information about the homogeneity of variances assumption. Hoekstra, Kiers, and Johnson (2012) have shown that from 50 randomly selected publications in *Psychological Science* that used at least one ANOVA, *t*-test, or regression, only three discussed the normality or heterogeneity of variances assumptions. To generalize this result, we surveyed statistical tests reported in 116 articles in the *Journal of Personality and Social Psychology* published in the year 2016. In 14% of these articles a One-Way ANOVA was reported, but none of the articles explicitly reported a homogeneity check, and information regarding the homogeneity of variances (e.g. the corrected degrees of freedom due to unequal variances) was observed in only one article. Despite the fact that the *F-*test is currently used by default, alternatives such as the *W-*test are often a better choice, given that the test has nearly the same statistical power, but provides better Type 1 error control than the *F*-test when variances are unequal. Moreover, the *W*-test is available in practically all statistical software packages. R and Minitab present the *W*-test by default: Users can request the *F*-test, but only after explicitly stating that the assumption of equal variances is met (see the section “Conducting Shapiro-Wilk and *W*-test in R or SPSS”).

In this paper, we review the differences between the *F*-test, *W*-test and *F\**-test. Based on extensive simulations that compare the Type 1 error rate and the statistical power of these three tests, we suggest that the *W*-test is a better default for psychological science than the *F*-test and *F\**-test when comparing the means of groups. There are other alternative tests than the ones we examine in this manuscript, but these tests have a different null hypothesis than the equality of means, such as the trimmed means test and nonparametric tests (e.g. the Kruskal-Wallis test and the Mann-Whitney U test)[[1]](#endnote-1). All these tests can be very useful in situations where distributions should be compared based on other parameters than means (e.g. when means are not a good representation of a distribution; Hayes & Cai, 2007). However, since most hypotheses in psychology are comparing means, we limit our analysis to these tests in order to highlight two main points. First, there are situations where tests comparing means will give invalid results and should be avoided. Second, when comparisons of means provide valid results, researchers can improve their statistical inferences by replacing the *F*-test with *W*-test.[[2]](#endnote-2)

All parametric tests rely on assumptions about the data. While the *F*-test relies on the normality and homogeneity of variances, other alternatives (e.g. *W*-test) only depend on the normality assumption. When both normality and homogeneity of variances are met, the *F*-test is slightly more powerful than alternatives. However, the assumptions of normality and homogeneity of variances are rarely met in real life (Erceg-Hurn & Mirosevich, 2008; Fan & Hancock, 2012). Particularly when groups are extracted from populations that have unequal variances, the *F*-test can be severely biased and lead to invalid statistical inferences[[3]](#endnote-3) (i.e., incorrect Type 1 error rates and deviations from the desired power). When comparing only two groups, the problem of unequal variances can be dealt through experimental design (i.e., collecting the same number of participants in each group). However, when comparing more than two groups, the *F*-test with unequal variances is even too liberal when sample sizes are equal across groups (Box, 1954), the problem of unequal variances can only be dealt with by choosing the correct statistical test.

Here, we argue that there are no strong reasons to assume by default that the equal variances assumption is met in psychological research, nor that there are substantial costs in abandoning this assumption.

We first explain why the assumptions of normality and equal variances are not always plausible in psychology, and provide examples of research areas where unequal variances should be expected. We will then review differences between the *F*-test, *W*-test and *F\**-test and show through simulations that unequal variances between groups have a larger impact on Type 1 error rate and power than normality violations. We will argue that the Type 1 error inflation observed with the *F*-test or *F\**-test when variances are unequal is much more problematic than the possible small loss of statistical power when the *W*-test is used when variances are equal. Finally, we will point out cases where the *W*-test is not recommended. As we will show, the test is not robust against departures from the normality assumption, when sample sizes are small (i.e., n < 50). We provide recommendations to detect and deal with these situations.

# Why You Should Think About the Assumptions Underlying Parametric Tests

When the assumptions of parametric tests are not met, the conclusions based on parametric tests can be severely biased (Lix et al., 1996), both in terms of Type 1 error rate and power. Notwithstanding, it seems that researchers rarely take the assumptions of parametric tests into consideration before performing an ANOVA, or at the very least they rarely discuss assumption checks.

When researchers do check for assumptions, they often follow a two-step procedure that is recommended in many textbooks (Field, 2013; Howell, 2012). As a first step, researchers are recommended to statistically and/or visually examine the assumptions of normality and equal variances (or the homoscedasticity assumption), before in the second step performing a suitable statistical test (Delacre et al., 2017). However, this two-step procedure is not recommended. Several authors have shown the limitations of conducting such a procedure when comparing two groups (Rasch, Kubinger, & Moder, 2011; Ruxton, 2006; Zimmerman, 2004), and these limitations remain true when applying other tests that assume equality of variances, such as *F*-test or regression (Wilcox, Granger, & Clark., 2013).

# Assumption tests for normality can lack power to detect deviations from normality. For example, while the Kolmogorov-Smirnov test is very often used, it is not recommended as an assumption check (Ghasemi & Zahediasl, 2012) because it will often fail to detect differences between the normal distribution and other distributions (Ghasemi & Zahediasl, 2012; Thode, 2002; Wilcox, 2005), such as the normal skewed distribution (see Supplemental Material 1). Instead, the Shapiro-Wilk test (available in SPSS)[[4]](#endnote-4) is recommended because it is more powerful (Ghasemi & Zahediasl, 2012;

## **Supplemental Material 4: power of the *F*-test, *W*-test and *F\**-test**

Assuming the null hypothesis is false and a Type 1 error rate of 5%, a test can yield either a significant result (*p*-value < 5%; or a “true positive” -TP) or a non-significant result (*p*-value > 5%; or a “false negative”-FN). The power is the relative frequency of effects detected as significant, when the null is false (i.e. when there is real differences between groups):

Power= =

In order to compute the power of the *F*-test and 2 famous alternatives when population variances are unequal (*W*-test and *F\**-test of comparison of means, both available on SPSS), we performed 1,000,000 simulations of k samples (where k is respectively 2 and 3)[[5]](#footnote-1) generated under 560 conditions (yielding 2\*560,000,000 simulations in total).

In each condition, k-1 samples were generated from a population where and sample size was 20,30,40,50 or 100. The standard deviation and the sample size of the last sample is a function of the sample sizes ratio (n-ratio = ; ranging from 0.5 to 2 in steps of 0.5) and the SDR (0.5,1,2 or 4). In all conditions, the mean of k-1 groups was 0 and the mean of the last group was 1. Note that because standard deviations and mean deviations vary from one condition to another, the effect size is not systematically the same in all conditions.

The set of simulations was repeated seven times varying the distributions underlying the data.

* **k normal distributions**: In order to assess the power of all tests when the normality assumption is met, data were generated by means of the function “rnorm” (from the package “stats”; “R: The Normal Distribution,” 2016) . Results are in Table A4.1a and A4.1b.
* **k double exponential distributions**: In order to assess the impact of high kurtosis on the power of all tests, data were generated by means of the function “rdoublex” (from the package “smoothmest”; "R: The double exponential (Laplace) distribution," 2012). Results are in Table A4.2a and A4.2b.
* **k mixed normal distributions**: In order to assess the impact of extremely high kurtosis on the power of all tests, regardless of variance, data were generated by means of the function “rmixnorm” (from the package “bda”; Wang & Wang, 2015). Results are in Table A4.3a and A4.3b.
* **k normal skewed distributions with positive skewness of +0.99**: In order to assess the impact of moderate skewness on the power of all tests, data were generated by means of the function “rsnorm” (from the package “fGarch”; “R: Skew Normal Distribution,” 2017). The normal skewed distribution was chosen because it is the only skewed distribution where the standard deviation ratio can vary without having an impact on skewness. Results are in Table A4.4a and A4.4b.
* **k-1 normal skewed distributions with positive skewness of +0.99** **and 1 normal skewed distribution with negative skewness of -0.99**: In order to assess the impact of unequal shapes, in terms of skewness, on the power of all tests, when data have moderate skewness, data were generated by means of the functions “rsnorm” (from the package “fGarch”; “R: Skew Normal Distribution,” 2017). Results are in Table A4.5a and A4.5b.
* **k-1 chi square distributions with two degrees of freedom, and one normal skewed distribution with positive skewness of +0.99**: In order to assess the impact of high asymetry on the power of all tests, k-1 distributions were generated by means of the functions “rchisq” (“R: The (non-central) Chi-Squared Distribution,” 2016). Because the chi square is non-negative, it is not possible to generate chi-square where = 1, 4 or 8 and µi is the same than the chi-square with two degrees of freedom. However, we wanted to assess the impact of different SDR on type 1 error rate. For these reasons, the kth distribution was generated by means of “rsnorm” in order to follow a normal skewed distribution with positive skewness of +0.99 and mean = 2 (from the package “fGarch”; “R: Skew Normal Distribution,” 2017). Results are in Table A4.6a and A4.6b.
* **k-1 chi square distributions with two degrees of freedom, and one normal skewed distribution with negative skewness of -0.99**: In order to assess the impact of unequal shapes, in terms of skewness, on power of all tests, when distributions have extreme skewness, k-1 distributions were generated by means of the functions “rchisq” (“R: The (non-central) Chi-Squared Distribution,” 2016). The kth distribution was generated by means of “rsnorm” in order to follow a normal skewed distribution with positive skewness of +0.99 and mean = 2 (from the package “fGarch”; “R: Skew Normal Distribution,” 2017). Results are in Table A4.7a and A4.7b.

Finally, because of a common confusion between kurtosis and variance (DeCarlo, 1997, see Supplemental Material 3), and in order to show the impact of kurtosis on power, independently of the variance, a last set of simulations was created and in each condition, k-1 samples were generated from a double exponential distribution where β were 2 (i.e. j2.82) and sample sizes ( were 20,30,40,50 or 100. The scale parameter β and the sample size of the kth group was a function of the sample sizes ratio (n-ratio = ; ranging from 0.5 to 2 in steps of 0.5) and the SDR (respectively 0.5,1,2 or 4). Results are in Table A4.8a and A4.8b.

The observed power was computed by repeating two steps for each condition: in a first step, the *p*-values of the *F*-test, *W*-test and *F*\*-test were extracted for eah dataset, and in a second step, the percent of *p*-values under the nominal alpha risk (5%) was computed for each condition and for each test. We used R commands to generate data from different distributions.

In order to insure the reliability of our calculation method, the observed power, computed when data were extracted from normal distributions (see Table A4.1a and A4.1b), was compared with theoretical power, i.e. the power computed using the power function of the *W*-test, *F*-test and *F\**-test. When assumptions underlying each test are met (i.e. normality for all tests, and equal variances for *F*-test), the computed power is very consistent with theoretical power, one can therefore conclude that the method is reliable.

**Results of the F-test.** When the normality assumption assumptions is met, but the homoscedasticity assumption is not, the power of the *F*-test is not consistent with theoretical expectations. It is particularly true with unequal sample sizes between groups: when there is a positive correlation between sample sizes and standard deviations, power is smaller than expectations, meaning that the power-curve will conduct to overestimate the real power (even when there are 100 subjects per groups). On the other side, when there is a negative correlation between n and sd, power is bigger than expectations, meaning that the power-curve will conduct to underestimate the real power (even when there are 100 subjects per groups). Finally, with equal sample sizes between groups and unequal variances, the power curve with either underestimate the real power (with small sample sizes; i.e ni=20) or overestimate the real power (with big sample sizes; i.e. ni=100).

When the assumption of equal variances is met, one obtains a gain in power especially when distributions have a big kurtosis (See Table A4.3a and A4.3b), or when high skewnesses are combined with skewnesses of opposite signs (See Tables A4.7a and A4.7b). However, the bigger are sample sizes, the closest is the power from the power in normal cases. For examples, with 50 subjets per group, deviations between the observed power and the expected power decreases, whatever distributions the data are extracted from (See Tables A4.3a, A4.3b, A4.7a and A4.7b). Finally, when the assumption of equal variances is not met, the effect of high kurtosis can become bigger than where variances are equal between groups.

## **Results of the W-test**. When the normality assumption is met, but the homoscedasticity assumption is not, contrary to what was observed for *F*-test, the power of the *W*-test is very consistent with theoretical expectations, because the *W*-test is robust against homoscedasticity violations.

However, the *W*-test is in general more affected by abnormality violations than *F*-test, except when homoscedasticity is combined with equal sample sizes and only two groups to compare. In all other situations, there is a bigger gain in power with *W*-test than with *F*-test, particularly when distributions have a high kurtosis (A4.3a and A4.3b), or are highly skewed with unequal skewnesses between groups (see Table A4.7a and A4.7b). Moreover, the gain in power is more important when sample sizes are unequal between groups. However, the bigger are sample sizes, the closest is the power from the power in normal cases.

## **Results of the *F\**-test.** When there are only two groups to compare, the *F\**-test and *W*-test are identical. The power of *F\**-test is therefore very consistent with theoretical expectations, even when variances are unequal between groups. However, when there are more than two groups to compare and unequal variances between groups, power of the *F\**-test is more consistent with theoretical expectations than power of the *F*-test, but less consistent than power of the *W*-test. Whatever the correlation between sample sizes and standard deviations (positive, negative or null), power is bigger than expectations, meaning that the power-curve will conduct to underestimate the real power. This is particularly true with small sample sizes. When sample sizes increase, the gain in power decreases (and one observes a power lower than expectations, even when there are 100 subjects per groups).

Moreover, with heavy tailed distributions, there is a gain in power in comparison with normal distributions. When distribution are skewed, there is either a gain or a loss in power, in comparison with normal distributions, depending on the highness of skewness, and if skewness are of same or opposite signs between groups.

Finally, when the assumption of equal variances is not met, the *F\**-test is more affected by abnomality violations than the *F*- test, but less affected by abnormality violations than the *W*-test. However, as both other tests, the effect of skewed distributions becomes bigger than where variances are equal between groups, and depends on the situation (either a gain or a loss in power).

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| Table A4.1a  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared and samples are extracted from normal distributions.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,299** | **0,239** | **0,144** | **0,081** | **0,422** | **0,231** | **0,105** | **0,064** | **0,422** | **0,231** | **0,105** | **0,064** |
|  | |  | Obs. | 0,265 | 0,239 | 0,206 | 0,192 | 0,419 | 0,232 | 0,107 | 0,065 | 0,419 | 0,232 | 0,107 | 0,065 |
| **20** | | **1** | **Theo.** | **0,496** | **0,338** | **0,164** | **0,083** | **0,489** | **0,338** | **0,162** | **0,081** | **0,489** | **0,338** | **0,162** | **0,081** |
|  | |  | Obs. | 0,498 | 0,338 | 0,168 | 0,089 | 0,489 | 0,337 | 0,162 | 0,082 | 0,489 | 0,337 | 0,162 | 0,082 |
| **20** | | **1.5** | **Theo.** | **0,631** | **0,397** | **0,173** | **0,083** | **0,513** | **0,394** | **0,208** | **0,098** | **0,513** | **0,394** | **0,208** | **0,098** |
|  | |  | Obs. | 0,618 | 0,397 | 0,146 | 0,050 | 0,513 | 0,394 | 0,207 | 0,097 | 0,513 | 0,394 | 0,207 | 0,097 |
| **20** | | **2** | **Theo.** | **0,722** | **0,435** | **0,179** | **0,084** | **0,525** | **0,429** | **0,246** | **0,113** | **0,525** | **0,429** | **0,246** | **0,113** |
|  | |  | Obs. | 0,686 | 0,435 | 0,131 | 0,031 | 0,525 | 0,428 | 0,245 | 0,113 | 0,525 | 0,428 | 0,245 | 0,113 |
| **30** | | **0.5** | **Theo.** | **0,428** | **0,340** | **0,196** | **0,098** | **0,589** | **0,333** | **0,139** | **0,073** | **0,589** | **0,333** | **0,139** | **0,073** |
|  | |  | Obs. | 0,414 | 0,339 | 0,250 | 0,201 | 0,588 | 0,332 | 0,140 | 0,073 | 0,588 | 0,332 | 0,140 | 0,073 |
| **30** | | **1** | **Theo.** | **0,673** | **0,478** | **0,226** | **0,100** | **0,668** | **0,478** | **0,224** | **0,099** | **0,668** | **0,478** | **0,224** | **0,099** |
|  | |  | Obs. | 0,673 | 0,478 | 0,227 | 0,105 | 0,667 | 0,478 | 0,223 | 0,099 | 0,667 | 0,478 | 0,223 | 0,099 |
| **30** | | **1.5** | **Theo.** | **0,807** | **0,553** | **0,239** | **0,101** | **0,696** | **0,551** | **0,292** | **0,123** | **0,696** | **0,551** | **0,292** | **0,123** |
|  | |  | Obs. | 0,778 | 0,553 | 0,214 | 0,064 | 0,695 | 0,551 | 0,291 | 0,124 | 0,695 | 0,551 | 0,291 | 0,124 |
| **30** | | **2** | **Theo.** | **0,880** | **0,599** | **0,247** | **0,102** | **0,710** | **0,594** | **0,346** | **0,147** | **0,710** | **0,594** | **0,346** | **0,147** |
|  | |  | Obs. | 0,832 | 0,600 | 0,203 | 0,043 | 0,710 | 0,594 | 0,347 | 0,147 | 0,710 | 0,594 | 0,347 | 0,147 |
| **40** | | **0.5** | **Theo.** | **0,543** | **0,435** | **0,247** | **0,114** | **0,719** | **0,429** | **0,173** | **0,082** | **0,719** | **0,429** | **0,173** | **0,082** |
|  | |  | Obs. | 0,553 | 0,435 | 0,294 | 0,214 | 0,718 | 0,429 | 0,173 | 0,082 | 0,718 | 0,429 | 0,173 | 0,082 |
| **40** | | **1** | **Theo.** | **0,798** | **0,598** | **0,287** | **0,118** | **0,794** | **0,598** | **0,285** | **0,117** | **0,794** | **0,598** | **0,285** | **0,117** |
|  | |  | Obs. | 0,797 | 0,599 | 0,288 | 0,121 | 0,794 | 0,598 | 0,285 | 0,116 | 0,794 | 0,598 | 0,285 | 0,116 |
| **40** | | **1.5** | **Theo.** | **0,906** | **0,679** | **0,305** | **0,119** | **0,819** | **0,678** | **0,372** | **0,150** | **0,819** | **0,678** | **0,372** | **0,150** |
|  | |  | Obs. | 0,879 | 0,679 | 0,284 | 0,080 | 0,820 | 0,677 | 0,373 | 0,149 | 0,820 | 0,677 | 0,373 | 0,149 |
| **40** | | **2** | **Theo.** | **0,952** | **0,726** | **0,315** | **0,120** | **0,832** | **0,722** | **0,441** | **0,181** | **0,832** | **0,722** | **0,441** | **0,181** |
|  | |  | Obs. | 0,914 | 0,726 | 0,278 | 0,056 | 0,833 | 0,722 | 0,440 | 0,181 | 0,833 | 0,722 | 0,440 | 0,181 |
| **50** | | **0.5** | **Theo.** | **0,641** | **0,522** | **0,298** | **0,131** | **0,813** | **0,516** | **0,207** | **0,091** | **0,813** | **0,516** | **0,207** | **0,091** |
|  | |  | Obs. | 0,670 | 0,522 | 0,337 | 0,226 | 0,813 | 0,516 | 0,208 | 0,091 | 0,813 | 0,516 | 0,208 | 0,091 |
| **50** | | **1** | **Theo.** | **0,879** | **0,697** | **0,347** | **0,136** | **0,877** | **0,697** | **0,345** | **0,134** | **0,877** | **0,697** | **0,345** | **0,134** |
|  | |  | Obs. | 0,879 | 0,697 | 0,348 | 0,139 | 0,877 | 0,697 | 0,345 | 0,135 | 0,877 | 0,697 | 0,345 | 0,135 |
| **50** | | **1.5** | **Theo.** | **0,956** | **0,776** | **0,368** | **0,138** | **0,897** | **0,774** | **0,449** | **0,176** | **0,897** | **0,774** | **0,449** | **0,176** |
|  | |  | Obs. | 0,935 | 0,776 | 0,353 | 0,097 | 0,897 | 0,775 | 0,449 | 0,176 | 0,897 | 0,775 | 0,449 | 0,176 |
| **50** | | **2** | **Theo.** | **0,982** | **0,818** | **0,380** | **0,139** | **0,907** | **0,816** | **0,527** | **0,215** | **0,907** | **0,816** | **0,527** | **0,215** |
|  | |  | Obs. | 0,957 | 0,818 | 0,356 | 0,071 | 0,906 | 0,815 | 0,527 | 0,215 | 0,906 | 0,815 | 0,527 | 0,215 |
| **100** | | **0.5** | **Theo.** | **0,911** | **0,818** | **0,529** | **0,217** | **0,982** | **0,816** | **0,375** | **0,137** | **0,982** | **0,816** | **0,375** | **0,137** |
|  | |  | Obs. | 0,950 | 0,818 | 0,525 | 0,294 | 0,982 | 0,815 | 0,375 | 0,137 | 0,982 | 0,815 | 0,375 | 0,137 |
| **100** | | **1** | **Theo.** | **0,994** | **0,940** | **0,605** | **0,226** | **0,993** | **0,940** | **0,603** | **0,225** | **0,993** | **0,940** | **0,603** | **0,225** |
|  | |  | Obs. | 0,994 | 0,941 | 0,604 | 0,228 | 0,994 | 0,941 | 0,603 | 0,225 | 0,994 | 0,941 | 0,603 | 0,225 |
| **100** | | **1.5** | **Theo.** | **0,999** | **0,971** | **0,635** | **0,230** | **0,996** | **0,971** | **0,739** | **0,307** | **0,996** | **0,971** | **0,739** | **0,307** |
|  | |  | Obs. | 0,998 | 0,971 | 0,651 | 0,188 | 0,996 | 0,971 | 0,739 | 0,307 | 0,996 | 0,971 | 0,739 | 0,307 |
| **100** | | **2** | **Theo.** | **1,000** | **0,983** | **0,651** | **0,232** | **0,997** | **0,982** | **0,821** | **0,382** | **0,997** | **0,982** | **0,821** | **0,382** |
|  | |  | Obs. | 0,999 | 0,982 | 0,683 | 0,162 | 0,997 | 0,982 | 0,820 | 0,382 | 0,997 | 0,982 | 0,820 | 0,382 |

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| Table A4.1b  *Comparison between observed and expected power, when nominal alpha risk = 5%, three groups are compared and samples are extracted from normal distributions.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,244** | **0,214** | **0,150** | **0,088** | **0,463** | **0,204** | **0,090** | **0,060** | **0,291** | **0,211** | **0,112** | **0,067** |
|  | |  | Obs. | 0,192 | 0,214 | 0,220 | 0,232 | 0,463 | 0,209 | 0,095 | 0,062 | 0,282 | 0,209 | 0,124 | 0,086 |
| **20** | | **1** | **Theo.** | **0,435** | **0,337** | **0,186** | **0,092** | **0,591** | **0,329** | **0,135** | **0,072** | **0,431** | **0,337** | **0,182** | **0,089** |
|  | |  | Obs. | 0,452 | 0,338 | 0,196 | 0,124 | 0,593 | 0,329 | 0,136 | 0,073 | 0,443 | 0,336 | 0,188 | 0,111 |
| **20** | | **1.5** | **Theo.** | **0,589** | **0,424** | **0,205** | **0,094** | **0,644** | **0,414** | **0,176** | **0,083** | **0,512** | **0,423** | **0,244** | **0,111** |
|  | |  | Obs. | 0,627 | 0,424 | 0,181 | 0,076 | 0,647 | 0,413 | 0,176 | 0,083 | 0,532 | 0,421 | 0,245 | 0,132 |
| **20** | | **2** | **Theo.** | **0,703** | **0,487** | **0,217** | **0,095** | **0,672** | **0,473** | **0,214** | **0,094** | **0,565** | **0,483** | **0,297** | **0,132** |
|  | |  | Obs. | 0,732 | 0,487 | 0,169 | 0,050 | 0,677 | 0,472 | 0,213 | 0,093 | 0,586 | 0,481 | 0,292 | 0,151 |
| **30** | | **0.5** | **Theo.** | **0,357** | **0,310** | **0,208** | **0,109** | **0,658** | **0,300** | **0,116** | **0,066** | **0,425** | **0,307** | **0,152** | **0,078** |
|  | |  | Obs. | 0,320 | 0,310 | 0,267 | 0,242 | 0,657 | 0,303 | 0,118 | 0,068 | 0,440 | 0,305 | 0,162 | 0,099 |
| **30** | | **1** | **Theo.** | **0,617** | **0,489** | **0,265** | **0,116** | **0,791** | **0,482** | **0,186** | **0,084** | **0,613** | **0,489** | **0,261** | **0,112** |
|  | |  | Obs. | 0,659 | 0,490 | 0,267 | 0,143 | 0,793 | 0,482 | 0,187 | 0,085 | 0,654 | 0,489 | 0,261 | 0,133 |
| **30** | | **1.5** | **Theo.** | **0,782** | **0,602** | **0,295** | **0,118** | **0,838** | **0,594** | **0,251** | **0,102** | **0,708** | **0,601** | **0,355** | **0,147** |
|  | |  | Obs. | 0,817 | 0,602 | 0,263 | 0,098 | 0,841 | 0,594 | 0,251 | 0,102 | 0,748 | 0,600 | 0,341 | 0,165 |
| **30** | | **2** | **Theo.** | **0,876** | **0,676** | **0,313** | **0,120** | **0,861** | **0,665** | **0,310** | **0,119** | **0,762** | **0,673** | **0,431** | **0,180** |
|  | |  | Obs. | 0,889 | 0,675 | 0,260 | 0,070 | 0,864 | 0,666 | 0,309 | 0,119 | 0,797 | 0,671 | 0,410 | 0,194 |
| **40** | | **0.5** | **Theo.** | **0,464** | **0,404** | **0,268** | **0,131** | **0,796** | **0,394** | **0,142** | **0,072** | **0,548** | **0,401** | **0,194** | **0,090** |
|  | |  | Obs. | 0,463 | 0,404 | 0,315 | 0,256 | 0,796 | 0,395 | 0,145 | 0,074 | 0,591 | 0,400 | 0,202 | 0,112 |
| **40** | | **1** | **Theo.** | **0,754** | **0,621** | **0,345** | **0,140** | **0,903** | **0,615** | **0,238** | **0,096** | **0,751** | **0,621** | **0,341** | **0,137** |
|  | |  | Obs. | 0,808 | 0,621 | 0,334 | 0,164 | 0,904 | 0,615 | 0,239 | 0,097 | 0,805 | 0,621 | 0,330 | 0,156 |
| **40** | | **1.5** | **Theo.** | **0,893** | **0,739** | **0,384** | **0,144** | **0,934** | **0,733** | **0,327** | **0,121** | **0,837** | **0,738** | **0,461** | **0,184** |
|  | |  | Obs. | 0,919 | 0,740 | 0,346 | 0,119 | 0,935 | 0,734 | 0,326 | 0,121 | 0,878 | 0,739 | 0,432 | 0,198 |
| **40** | | **2** | **Theo.** | **0,954** | **0,808** | **0,407** | **0,146** | **0,947** | **0,800** | **0,405** | **0,146** | **0,881** | **0,805** | **0,553** | **0,230** |
|  | |  | Obs. | 0,958 | 0,808 | 0,353 | 0,091 | 0,949 | 0,802 | 0,404 | 0,145 | 0,912 | 0,805 | 0,516 | 0,236 |
| **50** | | **0.5** | **Theo.** | **0,563** | **0,493** | **0,328** | **0,154** | **0,884** | **0,484** | **0,168** | **0,078** | **0,653** | **0,491** | **0,237** | **0,102** |
|  | |  | Obs. | 0,595 | 0,492 | 0,358 | 0,270 | 0,884 | 0,484 | 0,171 | 0,080 | 0,716 | 0,489 | 0,240 | 0,124 |
| **50** | | **1** | **Theo.** | **0,849** | **0,727** | **0,423** | **0,165** | **0,958** | **0,722** | **0,292** | **0,109** | **0,847** | **0,727** | **0,419** | **0,162** |
|  | |  | Obs. | 0,900 | 0,727 | 0,400 | 0,186 | 0,959 | 0,722 | 0,293 | 0,110 | 0,899 | 0,726 | 0,396 | 0,179 |
| **50** | | **1.5** | **Theo.** | **0,951** | **0,836** | **0,469** | **0,170** | **0,975** | **0,832** | **0,401** | **0,141** | **0,914** | **0,835** | **0,557** | **0,222** |
|  | |  | Obs. | 0,967 | 0,836 | 0,426 | 0,142 | 0,976 | 0,832 | 0,401 | 0,141 | 0,946 | 0,835 | 0,517 | 0,229 |
| **50** | | **2** | **Theo.** | **0,984** | **0,891** | **0,496** | **0,173** | **0,982** | **0,887** | **0,494** | **0,173** | **0,944** | **0,890** | **0,658** | **0,281** |
|  | |  | Obs. | 0,986 | 0,891 | 0,443 | 0,112 | 0,982 | 0,888 | 0,493 | 0,171 | 0,965 | 0,890 | 0,608 | 0,276 |
| **100** | | **0.5** | **Theo.** | **0,872** | **0,810** | **0,599** | **0,272** | **0,996** | **0,806** | **0,308** | **0,111** | **0,930** | **0,809** | **0,451** | **0,165** |
|  | |  | Obs. | 0,948 | 0,810 | 0,550 | 0,341 | 0,996 | 0,804 | 0,310 | 0,111 | 0,975 | 0,809 | 0,421 | 0,182 |
| **100** | | **1** | **Theo.** | **0,992** | **0,962** | **0,732** | **0,298** | **1,000** | **0,961** | **0,542** | **0,177** | **0,991** | **0,962** | **0,729** | **0,294** |
|  | |  | Obs. | 0,998 | 0,962 | 0,900 | 0,291 | 1,000 | 0,961 | 0,600 | 0,178 | 0,998 | 0,962 | 0,900 | 0,287 |
| **100** | | **1.5** | **Theo.** | **0,999** | **0,990** | **0,784** | **0,308** | **1,000** | **0,989** | **0,705** | **0,246** | **0,998** | **0,990** | **0,866** | **0,415** |
|  | |  | Obs. | 1,000 | 0,989 | 0,736 | 0,262 | 1,000 | 0,989 | 0,705 | 0,247 | 1,000 | 0,989 | 0,805 | 0,381 |
| **100** | | **2** | **Theo.** | **1,000** | **0,996** | **0,812** | **0,313** | **1,000** | **0,996** | **0,811** | **0,313** | **0,999** | **0,996** | **0,931** | **0,521** |
|  | |  | Obs. | 1,000 | 0,996 | 0,783 | 0,238 | 1,000 | 0,996 | 0,811 | 0,312 | 1,000 | 0,996 | 0,884 | 0,464 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.2a  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared and samples are extracted from double exponential distributions.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,265** | **0,239** | **0,206** | **0,192** | **0,419** | **0,232** | **0,107** | **0,065** | **0,419** | **0,232** | **0,107** | **0,065** |
|  | |  | Obs. | 0,298 | 0,258 | 0,216 | 0,196 | 0,445 | 0,257 | 0,118 | 0,064 | 0,445 | 0,257 | 0,118 | 0,064 |
| **20** | | **1** | **Theo.** | **0,498** | **0,338** | **0,168** | **0,089** | **0,489** | **0,337** | **0,162** | **0,082** | **0,489** | **0,337** | **0,162** | **0,082** |
|  | |  | Obs. | 0,524 | 0,358 | 0,180 | 0,092 | 0,515 | 0,356 | 0,175 | 0,084 | 0,515 | 0,356 | 0,175 | 0,084 |
| **20** | | **1.5** | **Theo.** | **0,618** | **0,397** | **0,146** | **0,050** | **0,513** | **0,394** | **0,207** | **0,097** | **0,513** | **0,394** | **0,207** | **0,097** |
|  | |  | Obs. | 0,635 | 0,414 | 0,158 | 0,052 | 0,540 | 0,413 | 0,219 | 0,102 | 0,540 | 0,413 | 0,219 | 0,102 |
| **20** | | **2** | **Theo.** | **0,686** | **0,435** | **0,131** | **0,031** | **0,525** | **0,428** | **0,245** | **0,113** | **0,525** | **0,428** | **0,245** | **0,113** |
|  | |  | Obs. | 0,698 | 0,449 | 0,142 | 0,033 | 0,555 | 0,449 | 0,256 | 0,118 | 0,555 | 0,449 | 0,256 | 0,118 |
| **30** | | **0.5** | **Theo.** | **0,414** | **0,339** | **0,250** | **0,201** | **0,588** | **0,332** | **0,140** | **0,073** | **0,588** | **0,332** | **0,140** | **0,073** |
|  | |  | Obs. | 0,439 | 0,357 | 0,263 | 0,208 | 0,602 | 0,358 | 0,155 | 0,075 | 0,602 | 0,358 | 0,155 | 0,075 |
| **30** | | **1** | **Theo.** | **0,673** | **0,478** | **0,227** | **0,105** | **0,667** | **0,478** | **0,223** | **0,099** | **0,667** | **0,478** | **0,223** | **0,099** |
|  | |  | Obs. | 0,684 | 0,491 | 0,240 | 0,109 | 0,679 | 0,490 | 0,235 | 0,103 | 0,679 | 0,490 | 0,235 | 0,103 |
| **30** | | **1.5** | **Theo.** | **0,778** | **0,553** | **0,214** | **0,064** | **0,695** | **0,551** | **0,291** | **0,124** | **0,695** | **0,551** | **0,291** | **0,124** |
|  | |  | Obs. | 0,783 | 0,562 | 0,225 | 0,067 | 0,706 | 0,561 | 0,302 | 0,128 | 0,706 | 0,561 | 0,302 | 0,128 |
| **30** | | **2** | **Theo.** | **0,832** | **0,600** | **0,203** | **0,043** | **0,710** | **0,594** | **0,347** | **0,147** | **0,710** | **0,594** | **0,347** | **0,147** |
|  | |  | Obs. | 0,833 | 0,607 | 0,213 | 0,045 | 0,720 | 0,605 | 0,355 | 0,152 | 0,720 | 0,605 | 0,355 | 0,152 |
| **40** | | **0.5** | **Theo.** | **0,553** | **0,435** | **0,294** | **0,214** | **0,718** | **0,429** | **0,173** | **0,082** | **0,718** | **0,429** | **0,173** | **0,082** |
|  | |  | Obs. | 0,568 | 0,450 | 0,307 | 0,221 | 0,724 | 0,449 | 0,191 | 0,085 | 0,724 | 0,449 | 0,191 | 0,085 |
| **40** | | **1** | **Theo.** | **0,797** | **0,599** | **0,288** | **0,121** | **0,794** | **0,598** | **0,285** | **0,116** | **0,794** | **0,598** | **0,285** | **0,116** |
|  | |  | Obs. | 0,799 | 0,607 | 0,302 | 0,127 | 0,795 | 0,606 | 0,298 | 0,122 | 0,795 | 0,606 | 0,298 | 0,122 |
| **40** | | **1.5** | **Theo.** | **0,879** | **0,679** | **0,284** | **0,080** | **0,820** | **0,677** | **0,373** | **0,149** | **0,820** | **0,677** | **0,373** | **0,149** |
|  | |  | Obs. | 0,876 | 0,684 | 0,294 | 0,084 | 0,820 | 0,683 | 0,381 | 0,155 | 0,820 | 0,683 | 0,381 | 0,155 |
| **40** | | **2** | **Theo.** | **0,914** | **0,726** | **0,278** | **0,056** | **0,833** | **0,722** | **0,440** | **0,181** | **0,833** | **0,722** | **0,440** | **0,181** |
|  | |  | Obs. | 0,911 | 0,729 | 0,288 | 0,058 | 0,832 | 0,726 | 0,447 | 0,185 | 0,832 | 0,726 | 0,447 | 0,185 |
| **50** | | **0.5** | **Theo.** | **0,670** | **0,522** | **0,337** | **0,226** | **0,813** | **0,516** | **0,208** | **0,091** | **0,813** | **0,516** | **0,208** | **0,091** |
|  | |  | Obs. | 0,678 | 0,533 | 0,349 | 0,235 | 0,813 | 0,532 | 0,224 | 0,096 | 0,813 | 0,532 | 0,224 | 0,096 |
| **50** | | **1** | **Theo.** | **0,879** | **0,697** | **0,348** | **0,139** | **0,877** | **0,697** | **0,345** | **0,135** | **0,877** | **0,697** | **0,345** | **0,135** |
|  | |  | Obs. | 0,876 | 0,702 | 0,360 | 0,143 | 0,874 | 0,702 | 0,357 | 0,139 | 0,874 | 0,702 | 0,357 | 0,139 |
| **50** | | **1.5** | **Theo.** | **0,935** | **0,776** | **0,353** | **0,097** | **0,897** | **0,775** | **0,449** | **0,176** | **0,897** | **0,775** | **0,449** | **0,176** |
|  | |  | Obs. | 0,931 | 0,776 | 0,363 | 0,100 | 0,893 | 0,775 | 0,456 | 0,181 | 0,893 | 0,775 | 0,456 | 0,181 |
| **50** | | **2** | **Theo.** | **0,957** | **0,818** | **0,356** | **0,071** | **0,906** | **0,815** | **0,527** | **0,215** | **0,906** | **0,815** | **0,527** | **0,215** |
|  | |  | Obs. | 0,954 | 0,818 | 0,363 | 0,074 | 0,902 | 0,815 | 0,533 | 0,221 | 0,902 | 0,815 | 0,533 | 0,221 |
| **100** | | **0.5** | **Theo.** | **0,950** | **0,818** | **0,525** | **0,294** | **0,982** | **0,815** | **0,375** | **0,137** | **0,982** | **0,815** | **0,375** | **0,137** |
|  | |  | Obs. | 0,946 | 0,818 | 0,532 | 0,300 | 0,980 | 0,815 | 0,389 | 0,143 | 0,980 | 0,815 | 0,389 | 0,143 |
| **100** | | **1** | **Theo.** | **0,994** | **0,941** | **0,604** | **0,228** | **0,994** | **0,941** | **0,603** | **0,225** | **0,994** | **0,941** | **0,603** | **0,225** |
|  | |  | Obs. | 0,992 | 0,939 | 0,610 | 0,233 | 0,992 | 0,939 | 0,608 | 0,230 | 0,992 | 0,939 | 0,608 | 0,230 |
| **100** | | **1.5** | **Theo.** | **0,998** | **0,971** | **0,651** | **0,188** | **0,996** | **0,971** | **0,739** | **0,307** | **0,996** | **0,971** | **0,739** | **0,307** |
|  | |  | Obs. | 0,997 | 0,970 | 0,654 | 0,194 | 0,995 | 0,969 | 0,740 | 0,312 | 0,995 | 0,969 | 0,740 | 0,312 |
| **100** | | **2** | **Theo.** | **0,999** | **0,982** | **0,683** | **0,162** | **0,997** | **0,982** | **0,820** | **0,382** | **0,997** | **0,982** | **0,820** | **0,382** |
|  | |  | Obs. | 0,999 | 0,981 | 0,683 | 0,165 | 0,995 | 0,981 | 0,820 | 0,386 | 0,995 | 0,981 | 0,820 | 0,386 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.2b  *Comparison between observed and expected power, when nominal alpha risk = 5%, three groups are compared and samples are extracted from double exponential distributions.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,192** | **0,214** | **0,220** | **0,232** | **0,463** | **0,209** | **0,095** | **0,062** | **0,282** | **0,209** | **0,124** | **0,086** |
|  | |  | Obs. | 0,210 | 0,224 | 0,227 | 0,236 | 0,502 | 0,241 | 0,103 | 0,059 | 0,300 | 0,222 | 0,133 | 0,084 |
| **20** | | **1** | **Theo.** | **0,452** | **0,338** | **0,196** | **0,124** | **0,593** | **0,329** | **0,136** | **0,073** | **0,443** | **0,336** | **0,188** | **0,111** |
|  | |  | Obs. | 0,473 | 0,352 | 0,208 | 0,127 | 0,625 | 0,356 | 0,146 | 0,072 | 0,462 | 0,349 | 0,200 | 0,114 |
| **20** | | **1.5** | **Theo.** | **0,627** | **0,424** | **0,181** | **0,076** | **0,647** | **0,413** | **0,176** | **0,083** | **0,532** | **0,421** | **0,245** | **0,132** |
|  | |  | Obs. | 0,642 | 0,439 | 0,193 | 0,080 | 0,680 | 0,439 | 0,187 | 0,084 | 0,550 | 0,434 | 0,255 | 0,137 |
| **20** | | **2** | **Theo.** | **0,732** | **0,487** | **0,169** | **0,050** | **0,677** | **0,472** | **0,213** | **0,093** | **0,586** | **0,481** | **0,292** | **0,151** |
|  | |  | Obs. | 0,741 | 0,499 | 0,180 | 0,053 | 0,710 | 0,498 | 0,224 | 0,096 | 0,602 | 0,493 | 0,301 | 0,157 |
| **30** | | **0.5** | **Theo.** | **0,320** | **0,310** | **0,267** | **0,242** | **0,657** | **0,303** | **0,118** | **0,068** | **0,440** | **0,305** | **0,162** | **0,099** |
|  | |  | Obs. | 0,339 | 0,321 | 0,277 | 0,249 | 0,678 | 0,337 | 0,131 | 0,067 | 0,455 | 0,321 | 0,175 | 0,102 |
| **30** | | **1** | **Theo.** | **0,659** | **0,490** | **0,267** | **0,143** | **0,793** | **0,482** | **0,187** | **0,085** | **0,654** | **0,489** | **0,261** | **0,133** |
|  | |  | Obs. | 0,668 | 0,500 | 0,279 | 0,149 | 0,806 | 0,503 | 0,200 | 0,086 | 0,662 | 0,498 | 0,273 | 0,139 |
| **30** | | **1.5** | **Theo.** | **0,817** | **0,602** | **0,263** | **0,098** | **0,841** | **0,594** | **0,251** | **0,102** | **0,748** | **0,600** | **0,341** | **0,165** |
|  | |  | Obs. | 0,817 | 0,610 | 0,275 | 0,102 | 0,851 | 0,611 | 0,262 | 0,104 | 0,751 | 0,607 | 0,351 | 0,171 |
| **30** | | **2** | **Theo.** | **0,889** | **0,675** | **0,260** | **0,070** | **0,864** | **0,666** | **0,309** | **0,119** | **0,797** | **0,671** | **0,410** | **0,194** |
|  | |  | Obs. | 0,888 | 0,680 | 0,271 | 0,073 | 0,873 | 0,681 | 0,321 | 0,122 | 0,798 | 0,676 | 0,416 | 0,199 |
| **40** | | **0.5** | **Theo.** | **0,463** | **0,404** | **0,315** | **0,256** | **0,796** | **0,395** | **0,145** | **0,074** | **0,591** | **0,400** | **0,202** | **0,112** |
|  | |  | Obs. | 0,476 | 0,414 | 0,323 | 0,263 | 0,804 | 0,426 | 0,158 | 0,075 | 0,600 | 0,415 | 0,215 | 0,116 |
| **40** | | **1** | **Theo.** | **0,808** | **0,621** | **0,334** | **0,164** | **0,904** | **0,615** | **0,239** | **0,097** | **0,805** | **0,621** | **0,330** | **0,156** |
|  | |  | Obs. | 0,807 | 0,627 | 0,345 | 0,170 | 0,907 | 0,628 | 0,252 | 0,099 | 0,804 | 0,626 | 0,341 | 0,162 |
| **40** | | **1.5** | **Theo.** | **0,919** | **0,740** | **0,346** | **0,119** | **0,935** | **0,734** | **0,326** | **0,121** | **0,878** | **0,739** | **0,432** | **0,198** |
|  | |  | Obs. | 0,916 | 0,741 | 0,357 | 0,124 | 0,937 | 0,743 | 0,339 | 0,125 | 0,876 | 0,740 | 0,441 | 0,203 |
| **40** | | **2** | **Theo.** | **0,958** | **0,808** | **0,353** | **0,091** | **0,949** | **0,802** | **0,404** | **0,145** | **0,912** | **0,805** | **0,516** | **0,236** |
|  | |  | Obs. | 0,956 | 0,808 | 0,363 | 0,095 | 0,950 | 0,810 | 0,415 | 0,149 | 0,907 | 0,806 | 0,521 | 0,241 |
| **50** | | **0.5** | **Theo.** | **0,595** | **0,492** | **0,358** | **0,270** | **0,884** | **0,484** | **0,171** | **0,080** | **0,716** | **0,489** | **0,240** | **0,124** |
|  | |  | Obs. | 0,603 | 0,501 | 0,368 | 0,278 | 0,885 | 0,510 | 0,186 | 0,082 | 0,720 | 0,502 | 0,255 | 0,130 |
| **50** | | **1** | **Theo.** | **0,900** | **0,727** | **0,400** | **0,186** | **0,959** | **0,722** | **0,293** | **0,110** | **0,899** | **0,726** | **0,396** | **0,179** |
|  | |  | Obs. | 0,896 | 0,730 | 0,411 | 0,192 | 0,958 | 0,730 | 0,307 | 0,113 | 0,895 | 0,729 | 0,407 | 0,185 |
| **50** | | **1.5** | **Theo.** | **0,967** | **0,836** | **0,426** | **0,142** | **0,976** | **0,832** | **0,401** | **0,141** | **0,946** | **0,835** | **0,517** | **0,229** |
|  | |  | Obs. | 0,964 | 0,835 | 0,435 | 0,148 | 0,975 | 0,836 | 0,412 | 0,145 | 0,942 | 0,834 | 0,523 | 0,235 |
| **50** | | **2** | **Theo.** | **0,986** | **0,891** | **0,443** | **0,112** | **0,982** | **0,888** | **0,493** | **0,171** | **0,965** | **0,890** | **0,608** | **0,276** |
|  | |  | Obs. | 0,984 | 0,890 | 0,451 | 0,117 | 0,982 | 0,891 | 0,502 | 0,177 | 0,961 | 0,888 | 0,611 | 0,282 |
| **100** | | **0.5** | **Theo.** | **0,948** | **0,810** | **0,550** | **0,341** | **0,996** | **0,804** | **0,310** | **0,111** | **0,975** | **0,809** | **0,421** | **0,182** |
|  | |  | Obs. | 0,944 | 0,811 | 0,557 | 0,348 | 0,995 | 0,807 | 0,325 | 0,116 | 0,972 | 0,810 | 0,433 | 0,189 |
| **100** | | **1** | **Theo.** | **0,998** | **0,962** | **0,900** | **0,291** | **1,000** | **0,961** | **0,600** | **0,178** | **0,998** | **0,962** | **0,900** | **0,287** |
|  | |  | Obs. | 0,998 | 0,960 | 0,671 | 0,298 | 1,000 | 0,959 | 0,550 | 0,183 | 0,998 | 0,960 | 0,669 | 0,293 |
| **100** | | **1.5** | **Theo.** | **1,000** | **0,989** | **0,736** | **0,262** | **1,000** | **0,989** | **0,705** | **0,247** | **1,000** | **0,989** | **0,805** | **0,381** |
|  | |  | Obs. | 1,000 | 0,989 | 0,738 | 0,267 | 1,000 | 0,989 | 0,709 | 0,251 | 0,999 | 0,989 | 0,806 | 0,386 |
| **100** | | **2** | **Theo.** | **1,000** | **0,996** | **0,783** | **0,238** | **1,000** | **0,996** | **0,811** | **0,312** | **1,000** | **0,996** | **0,884** | **0,464** |
|  | |  | Obs. | 1,000 | 0,996 | 0,783 | 0,243 | 1,000 | 0,996 | 0,812 | 0,317 | 1,000 | 0,996 | 0,883 | 0,468 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.3a  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared and samples are extracted from mixed normal distributions.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,265** | **0,239** | **0,206** | **0,192** | **0,419** | **0,232** | **0,107** | **0,065** | **0,419** | **0,232** | **0,107** | **0,065** |
|  | |  | Obs. | 0,357 | 0,298 | 0,231 | 0,195 | 0,496 | 0,299 | 0,130 | 0,064 | 0,496 | 0,299 | 0,130 | 0,064 |
| **20** | | **1** | **Theo.** | **0,498** | **0,338** | **0,168** | **0,089** | **0,489** | **0,337** | **0,162** | **0,082** | **0,489** | **0,337** | **0,162** | **0,082** |
|  | |  | Obs. | 0,572 | 0,399 | 0,204 | 0,097 | 0,563 | 0,395 | 0,197 | 0,089 | 0,563 | 0,395 | 0,197 | 0,089 |
| **20** | | **1.5** | **Theo.** | **0,618** | **0,397** | **0,146** | **0,050** | **0,513** | **0,394** | **0,207** | **0,097** | **0,513** | **0,394** | **0,207** | **0,097** |
|  | |  | Obs. | 0,668 | 0,449 | 0,182 | 0,056 | 0,591 | 0,450 | 0,244 | 0,109 | 0,591 | 0,450 | 0,244 | 0,109 |
| **20** | | **2** | **Theo.** | **0,686** | **0,435** | **0,131** | **0,031** | **0,525** | **0,428** | **0,245** | **0,113** | **0,525** | **0,428** | **0,245** | **0,113** |
|  | |  | Obs. | 0,722 | 0,478 | 0,164 | 0,036 | 0,608 | 0,488 | 0,280 | 0,126 | 0,608 | 0,488 | 0,280 | 0,126 |
| **30** | | **0.5** | **Theo.** | **0,414** | **0,339** | **0,250** | **0,201** | **0,588** | **0,332** | **0,140** | **0,073** | **0,588** | **0,332** | **0,140** | **0,073** |
|  | |  | Obs. | 0,486 | 0,394 | 0,285 | 0,214 | 0,629 | 0,403 | 0,176 | 0,078 | 0,629 | 0,403 | 0,176 | 0,078 |
| **30** | | **1** | **Theo.** | **0,673** | **0,478** | **0,227** | **0,105** | **0,667** | **0,478** | **0,223** | **0,099** | **0,667** | **0,478** | **0,223** | **0,099** |
|  | |  | Obs. | 0,705 | 0,520 | 0,269 | 0,117 | 0,700 | 0,518 | 0,264 | 0,111 | 0,700 | 0,518 | 0,264 | 0,111 |
| **30** | | **1.5** | **Theo.** | **0,778** | **0,553** | **0,214** | **0,064** | **0,695** | **0,551** | **0,291** | **0,124** | **0,695** | **0,551** | **0,291** | **0,124** |
|  | |  | Obs. | 0,791 | 0,584 | 0,250 | 0,074 | 0,726 | 0,586 | 0,324 | 0,139 | 0,726 | 0,586 | 0,324 | 0,139 |
| **30** | | **2** | **Theo.** | **0,832** | **0,600** | **0,203** | **0,043** | **0,710** | **0,594** | **0,347** | **0,147** | **0,710** | **0,594** | **0,347** | **0,147** |
|  | |  | Obs. | 0,836 | 0,623 | 0,236 | 0,050 | 0,739 | 0,628 | 0,375 | 0,163 | 0,739 | 0,628 | 0,375 | 0,163 |
| **40** | | **0.5** | **Theo.** | **0,553** | **0,435** | **0,294** | **0,214** | **0,718** | **0,429** | **0,173** | **0,082** | **0,718** | **0,429** | **0,173** | **0,082** |
|  | |  | Obs. | 0,596 | 0,479 | 0,332 | 0,232 | 0,733 | 0,489 | 0,218 | 0,091 | 0,733 | 0,489 | 0,218 | 0,091 |
| **40** | | **1** | **Theo.** | **0,797** | **0,599** | **0,288** | **0,121** | **0,794** | **0,598** | **0,285** | **0,116** | **0,794** | **0,598** | **0,285** | **0,116** |
|  | |  | Obs. | 0,801 | 0,624 | 0,327 | 0,138 | 0,798 | 0,623 | 0,324 | 0,133 | 0,798 | 0,623 | 0,324 | 0,133 |
| **40** | | **1.5** | **Theo.** | **0,879** | **0,679** | **0,284** | **0,080** | **0,820** | **0,677** | **0,373** | **0,149** | **0,820** | **0,677** | **0,373** | **0,149** |
|  | |  | Obs. | 0,872 | 0,693 | 0,317 | 0,092 | 0,820 | 0,693 | 0,401 | 0,167 | 0,820 | 0,693 | 0,401 | 0,167 |
| **40** | | **2** | **Theo.** | **0,914** | **0,726** | **0,278** | **0,056** | **0,833** | **0,722** | **0,440** | **0,181** | **0,833** | **0,722** | **0,440** | **0,181** |
|  | |  | Obs. | 0,906 | 0,734 | 0,309 | 0,065 | 0,831 | 0,734 | 0,464 | 0,197 | 0,831 | 0,734 | 0,464 | 0,197 |
| **50** | | **0.5** | **Theo.** | **0,670** | **0,522** | **0,337** | **0,226** | **0,813** | **0,516** | **0,208** | **0,091** | **0,813** | **0,516** | **0,208** | **0,091** |
|  | |  | Obs. | 0,692 | 0,554 | 0,374 | 0,247 | 0,813 | 0,561 | 0,256 | 0,102 | 0,813 | 0,561 | 0,256 | 0,102 |
| **50** | | **1** | **Theo.** | **0,879** | **0,697** | **0,348** | **0,139** | **0,877** | **0,697** | **0,345** | **0,135** | **0,877** | **0,697** | **0,345** | **0,135** |
|  | |  | Obs. | 0,871 | 0,709 | 0,383 | 0,156 | 0,869 | 0,708 | 0,379 | 0,152 | 0,869 | 0,708 | 0,379 | 0,152 |
| **50** | | **1.5** | **Theo.** | **0,935** | **0,776** | **0,353** | **0,097** | **0,897** | **0,775** | **0,449** | **0,176** | **0,897** | **0,775** | **0,449** | **0,176** |
|  | |  | Obs. | 0,924 | 0,780 | 0,383 | 0,110 | 0,886 | 0,779 | 0,472 | 0,193 | 0,886 | 0,779 | 0,472 | 0,193 |
| **50** | | **2** | **Theo.** | **0,957** | **0,818** | **0,356** | **0,071** | **0,906** | **0,815** | **0,527** | **0,215** | **0,906** | **0,815** | **0,527** | **0,215** |
|  | |  | Obs. | 0,947 | 0,818 | 0,382 | 0,081 | 0,894 | 0,816 | 0,544 | 0,231 | 0,894 | 0,816 | 0,544 | 0,231 |
| **100** | | **0.5** | **Theo.** | **0,950** | **0,818** | **0,525** | **0,294** | **0,982** | **0,815** | **0,375** | **0,137** | **0,982** | **0,815** | **0,375** | **0,137** |
|  | |  | Obs. | 0,938 | 0,818 | 0,549 | 0,315 | 0,975 | 0,816 | 0,418 | 0,156 | 0,975 | 0,816 | 0,418 | 0,156 |
| **100** | | **1** | **Theo.** | **0,994** | **0,941** | **0,604** | **0,228** | **0,994** | **0,941** | **0,603** | **0,225** | **0,994** | **0,941** | **0,603** | **0,225** |
|  | |  | Obs. | 0,989 | 0,934 | 0,620 | 0,246 | 0,989 | 0,934 | 0,618 | 0,243 | 0,989 | 0,934 | 0,618 | 0,243 |
| **100** | | **1.5** | **Theo.** | **0,998** | **0,971** | **0,651** | **0,188** | **0,996** | **0,971** | **0,739** | **0,307** | **0,996** | **0,971** | **0,739** | **0,307** |
|  | |  | Obs. | 0,996 | 0,966 | 0,659 | 0,204 | 0,992 | 0,965 | 0,742 | 0,321 | 0,992 | 0,965 | 0,742 | 0,321 |
| **100** | | **2** | **Theo.** | **0,999** | **0,982** | **0,683** | **0,162** | **0,997** | **0,982** | **0,820** | **0,382** | **0,997** | **0,982** | **0,820** | **0,382** |
|  | |  | Obs. | 0,998 | 0,979 | 0,688 | 0,174 | 0,993 | 0,977 | 0,820 | 0,393 | 0,993 | 0,977 | 0,820 | 0,393 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.3b  *Comparison between observed and expected power, when nominal alpha risk = 5%, three groups are compared and samples are extracted from mixed normal distributions.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,192** | **0,214** | **0,220** | **0,232** | **0,463** | **0,209** | **0,095** | **0,062** | **0,282** | **0,209** | **0,124** | **0,086** |
|  | |  | Obs. | 0,248 | 0,247 | 0,237 | 0,235 | 0,572 | 0,288 | 0,113 | 0,057 | 0,337 | 0,246 | 0,141 | 0,083 |
| **20** | | **1** | **Theo.** | **0,452** | **0,338** | **0,196** | **0,124** | **0,593** | **0,329** | **0,136** | **0,073** | **0,443** | **0,336** | **0,188** | **0,111** |
|  | |  | Obs. | 0,510 | 0,385 | 0,231 | 0,133 | 0,679 | 0,409 | 0,167 | 0,072 | 0,498 | 0,378 | 0,222 | 0,119 |
| **20** | | **1.5** | **Theo.** | **0,627** | **0,424** | **0,181** | **0,076** | **0,647** | **0,413** | **0,176** | **0,083** | **0,532** | **0,421** | **0,245** | **0,132** |
|  | |  | Obs. | 0,665 | 0,467 | 0,217 | 0,085 | 0,732 | 0,486 | 0,211 | 0,086 | 0,581 | 0,459 | 0,276 | 0,146 |
| **20** | | **2** | **Theo.** | **0,732** | **0,487** | **0,169** | **0,050** | **0,677** | **0,472** | **0,213** | **0,093** | **0,586** | **0,481** | **0,292** | **0,151** |
|  | |  | Obs. | 0,753 | 0,523 | 0,205 | 0,058 | 0,761 | 0,545 | 0,250 | 0,099 | 0,628 | 0,517 | 0,321 | 0,167 |
| **30** | | **0.5** | **Theo.** | **0,320** | **0,310** | **0,267** | **0,242** | **0,657** | **0,303** | **0,118** | **0,068** | **0,440** | **0,305** | **0,162** | **0,099** |
|  | |  | Obs. | 0,375 | 0,344 | 0,293 | 0,256 | 0,720 | 0,395 | 0,148 | 0,067 | 0,486 | 0,351 | 0,194 | 0,103 |
| **30** | | **1** | **Theo.** | **0,659** | **0,490** | **0,267** | **0,143** | **0,793** | **0,482** | **0,187** | **0,085** | **0,654** | **0,489** | **0,261** | **0,133** |
|  | |  | Obs. | 0,683 | 0,522 | 0,304 | 0,159 | 0,828 | 0,546 | 0,227 | 0,089 | 0,676 | 0,519 | 0,297 | 0,148 |
| **30** | | **1.5** | **Theo.** | **0,817** | **0,602** | **0,263** | **0,098** | **0,841** | **0,594** | **0,251** | **0,102** | **0,748** | **0,600** | **0,341** | **0,165** |
|  | |  | Obs. | 0,820 | 0,625 | 0,301 | 0,110 | 0,871 | 0,643 | 0,292 | 0,110 | 0,757 | 0,620 | 0,373 | 0,183 |
| **30** | | **2** | **Theo.** | **0,889** | **0,675** | **0,260** | **0,070** | **0,864** | **0,666** | **0,309** | **0,119** | **0,797** | **0,671** | **0,410** | **0,194** |
|  | |  | Obs. | 0,884 | 0,689 | 0,295 | 0,080 | 0,889 | 0,709 | 0,348 | 0,129 | 0,798 | 0,685 | 0,434 | 0,211 |
| **40** | | **0.5** | **Theo.** | **0,463** | **0,404** | **0,315** | **0,256** | **0,796** | **0,395** | **0,145** | **0,074** | **0,591** | **0,400** | **0,202** | **0,112** |
|  | |  | Obs. | 0,502 | 0,434 | 0,343 | 0,274 | 0,818 | 0,483 | 0,183 | 0,076 | 0,617 | 0,444 | 0,241 | 0,122 |
| **40** | | **1** | **Theo.** | **0,808** | **0,621** | **0,334** | **0,164** | **0,904** | **0,615** | **0,239** | **0,097** | **0,805** | **0,621** | **0,330** | **0,156** |
|  | |  | Obs. | 0,808 | 0,640 | 0,370 | 0,183 | 0,912 | 0,657 | 0,282 | 0,106 | 0,804 | 0,638 | 0,365 | 0,175 |
| **40** | | **1.5** | **Theo.** | **0,919** | **0,740** | **0,346** | **0,119** | **0,935** | **0,734** | **0,326** | **0,121** | **0,878** | **0,739** | **0,432** | **0,198** |
|  | |  | Obs. | 0,909 | 0,746 | 0,378 | 0,135 | 0,940 | 0,760 | 0,364 | 0,133 | 0,869 | 0,743 | 0,456 | 0,216 |
| **40** | | **2** | **Theo.** | **0,958** | **0,808** | **0,353** | **0,091** | **0,949** | **0,802** | **0,404** | **0,145** | **0,912** | **0,805** | **0,516** | **0,236** |
|  | |  | Obs. | 0,950 | 0,808 | 0,384 | 0,103 | 0,952 | 0,823 | 0,438 | 0,158 | 0,899 | 0,805 | 0,533 | 0,253 |
| **50** | | **0.5** | **Theo.** | **0,595** | **0,492** | **0,358** | **0,270** | **0,884** | **0,484** | **0,171** | **0,080** | **0,716** | **0,489** | **0,240** | **0,124** |
|  | |  | Obs. | 0,617 | 0,518 | 0,386 | 0,290 | 0,886 | 0,558 | 0,214 | 0,085 | 0,725 | 0,527 | 0,282 | 0,137 |
| **50** | | **1** | **Theo.** | **0,900** | **0,727** | **0,400** | **0,186** | **0,959** | **0,722** | **0,293** | **0,110** | **0,899** | **0,726** | **0,396** | **0,179** |
|  | |  | Obs. | 0,891 | 0,735 | 0,432 | 0,206 | 0,958 | 0,745 | 0,336 | 0,121 | 0,888 | 0,734 | 0,428 | 0,199 |
| **50** | | **1.5** | **Theo.** | **0,967** | **0,836** | **0,426** | **0,142** | **0,976** | **0,832** | **0,401** | **0,141** | **0,946** | **0,835** | **0,517** | **0,229** |
|  | |  | Obs. | 0,958 | 0,834 | 0,454 | 0,159 | 0,974 | 0,843 | 0,436 | 0,154 | 0,934 | 0,832 | 0,535 | 0,248 |
| **50** | | **2** | **Theo.** | **0,986** | **0,891** | **0,443** | **0,112** | **0,982** | **0,888** | **0,493** | **0,171** | **0,965** | **0,890** | **0,608** | **0,276** |
|  | |  | Obs. | 0,980 | 0,887 | 0,468 | 0,126 | 0,980 | 0,895 | 0,521 | 0,186 | 0,953 | 0,884 | 0,619 | 0,293 |
| **100** | | **0.5** | **Theo.** | **0,948** | **0,810** | **0,550** | **0,341** | **0,996** | **0,804** | **0,310** | **0,111** | **0,975** | **0,809** | **0,421** | **0,182** |
|  | |  | Obs. | 0,938 | 0,813 | 0,569 | 0,361 | 0,993 | 0,811 | 0,357 | 0,125 | 0,968 | 0,811 | 0,457 | 0,203 |
| **100** | | **1** | **Theo.** | **0,998** | **0,962** | **0,900** | **0,291** | **1,000** | **0,961** | **0,600** | **0,178** | **0,998** | **0,962** | **0,900** | **0,287** |
|  | |  | Obs. | 0,997 | 0,957 | 0,677 | 0,310 | 0,999 | 0,956 | 0,566 | 0,193 | 0,997 | 0,957 | 0,676 | 0,306 |
| **100** | | **1.5** | **Theo.** | **1,000** | **0,989** | **0,736** | **0,262** | **1,000** | **0,989** | **0,705** | **0,247** | **1,000** | **0,989** | **0,805** | **0,381** |
|  | |  | Obs. | 1,000 | 0,987 | 0,741 | 0,277 | 1,000 | 0,987 | 0,716 | 0,261 | 0,999 | 0,987 | 0,807 | 0,395 |
| **100** | | **2** | **Theo.** | **1,000** | **0,996** | **0,783** | **0,238** | **1,000** | **0,996** | **0,811** | **0,312** | **1,000** | **0,996** | **0,884** | **0,464** |
|  | |  | Obs. | 1,000 | 0,995 | 0,784 | 0,252 | 1,000 | 0,995 | 0,815 | 0,326 | 1,000 | 0,995 | 0,882 | 0,475 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.4a  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared and samples are extracted from normal right skewed distributions.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,265** | **0,239** | **0,206** | **0,192** | **0,419** | **0,232** | **0,107** | **0,065** | **0,419** | **0,232** | **0,107** | **0,065** |
|  | |  | Obs. | 0,303 | 0,242 | 0,177 | 0,173 | 0,435 | 0,200 | 0,070 | 0,054 | 0,435 | 0,200 | 0,070 | 0,054 |
| **20** | | **1** | **Theo.** | **0,498** | **0,338** | **0,168** | **0,089** | **0,489** | **0,337** | **0,162** | **0,082** | **0,489** | **0,337** | **0,162** | **0,082** |
|  | |  | Obs. | 0,510 | 0,343 | 0,138 | 0,067 | 0,502 | 0,342 | 0,131 | 0,060 | 0,502 | 0,342 | 0,131 | 0,060 |
| **20** | | **1.5** | **Theo.** | **0,618** | **0,397** | **0,146** | **0,050** | **0,513** | **0,394** | **0,207** | **0,097** | **0,513** | **0,394** | **0,207** | **0,097** |
|  | |  | Obs. | 0,617 | 0,405 | 0,118 | 0,032 | 0,524 | 0,408 | 0,189 | 0,075 | 0,524 | 0,408 | 0,189 | 0,075 |
| **20** | | **2** | **Theo.** | **0,686** | **0,435** | **0,131** | **0,031** | **0,525** | **0,428** | **0,245** | **0,113** | **0,525** | **0,428** | **0,245** | **0,113** |
|  | |  | Obs. | 0,680 | 0,444 | 0,105 | 0,017 | 0,535 | 0,444 | 0,238 | 0,091 | 0,535 | 0,444 | 0,238 | 0,091 |
| **30** | | **0.5** | **Theo.** | **0,414** | **0,339** | **0,250** | **0,201** | **0,588** | **0,332** | **0,140** | **0,073** | **0,588** | **0,332** | **0,140** | **0,073** |
|  | |  | Obs. | 0,432 | 0,340 | 0,223 | 0,180 | 0,591 | 0,313 | 0,098 | 0,055 | 0,591 | 0,313 | 0,098 | 0,055 |
| **30** | | **1** | **Theo.** | **0,673** | **0,478** | **0,227** | **0,105** | **0,667** | **0,478** | **0,223** | **0,099** | **0,667** | **0,478** | **0,223** | **0,099** |
|  | |  | Obs. | 0,664 | 0,481 | 0,202 | 0,080 | 0,658 | 0,481 | 0,196 | 0,075 | 0,658 | 0,481 | 0,196 | 0,075 |
| **30** | | **1.5** | **Theo.** | **0,778** | **0,553** | **0,214** | **0,064** | **0,695** | **0,551** | **0,291** | **0,124** | **0,695** | **0,551** | **0,291** | **0,124** |
|  | |  | Obs. | 0,763 | 0,559 | 0,189 | 0,043 | 0,681 | 0,555 | 0,278 | 0,100 | 0,681 | 0,555 | 0,278 | 0,100 |
| **30** | | **2** | **Theo.** | **0,832** | **0,600** | **0,203** | **0,043** | **0,710** | **0,594** | **0,347** | **0,147** | **0,710** | **0,594** | **0,347** | **0,147** |
|  | |  | Obs. | 0,815 | 0,606 | 0,180 | 0,026 | 0,693 | 0,595 | 0,342 | 0,125 | 0,693 | 0,595 | 0,342 | 0,125 |
| **40** | | **0.5** | **Theo.** | **0,553** | **0,435** | **0,294** | **0,214** | **0,718** | **0,429** | **0,173** | **0,082** | **0,718** | **0,429** | **0,173** | **0,082** |
|  | |  | Obs. | 0,553 | 0,434 | 0,271 | 0,191 | 0,714 | 0,419 | 0,134 | 0,060 | 0,714 | 0,419 | 0,134 | 0,060 |
| **40** | | **1** | **Theo.** | **0,797** | **0,599** | **0,288** | **0,121** | **0,794** | **0,598** | **0,285** | **0,116** | **0,794** | **0,598** | **0,285** | **0,116** |
|  | |  | Obs. | 0,778 | 0,601 | 0,267 | 0,096 | 0,775 | 0,601 | 0,263 | 0,091 | 0,775 | 0,601 | 0,263 | 0,091 |
| **40** | | **1.5** | **Theo.** | **0,879** | **0,679** | **0,284** | **0,080** | **0,820** | **0,677** | **0,373** | **0,149** | **0,820** | **0,677** | **0,373** | **0,149** |
|  | |  | Obs. | 0,860 | 0,682 | 0,264 | 0,058 | 0,797 | 0,674 | 0,363 | 0,127 | 0,797 | 0,674 | 0,363 | 0,127 |
| **40** | | **2** | **Theo.** | **0,914** | **0,726** | **0,278** | **0,056** | **0,833** | **0,722** | **0,440** | **0,181** | **0,833** | **0,722** | **0,440** | **0,181** |
|  | |  | Obs. | 0,897 | 0,729 | 0,261 | 0,037 | 0,806 | 0,714 | 0,438 | 0,161 | 0,806 | 0,714 | 0,438 | 0,161 |
| **50** | | **0.5** | **Theo.** | **0,670** | **0,522** | **0,337** | **0,226** | **0,813** | **0,516** | **0,208** | **0,091** | **0,813** | **0,516** | **0,208** | **0,091** |
|  | |  | Obs. | 0,659 | 0,520 | 0,317 | 0,206 | 0,805 | 0,515 | 0,170 | 0,067 | 0,805 | 0,515 | 0,170 | 0,067 |
| **50** | | **1** | **Theo.** | **0,879** | **0,697** | **0,348** | **0,139** | **0,877** | **0,697** | **0,345** | **0,135** | **0,877** | **0,697** | **0,345** | **0,135** |
|  | |  | Obs. | 0,859 | 0,697 | 0,332 | 0,113 | 0,857 | 0,697 | 0,328 | 0,109 | 0,857 | 0,697 | 0,328 | 0,109 |
| **50** | | **1.5** | **Theo.** | **0,935** | **0,776** | **0,353** | **0,097** | **0,897** | **0,775** | **0,449** | **0,176** | **0,897** | **0,775** | **0,449** | **0,176** |
|  | |  | Obs. | 0,919 | 0,777 | 0,339 | 0,074 | 0,874 | 0,768 | 0,444 | 0,154 | 0,874 | 0,768 | 0,444 | 0,154 |
| **50** | | **2** | **Theo.** | **0,957** | **0,818** | **0,356** | **0,071** | **0,906** | **0,815** | **0,527** | **0,215** | **0,906** | **0,815** | **0,527** | **0,215** |
|  | |  | Obs. | 0,944 | 0,819 | 0,345 | 0,051 | 0,882 | 0,803 | 0,528 | 0,197 | 0,882 | 0,803 | 0,528 | 0,197 |
| **100** | | **0.5** | **Theo.** | **0,950** | **0,818** | **0,525** | **0,294** | **0,982** | **0,815** | **0,375** | **0,137** | **0,982** | **0,815** | **0,375** | **0,137** |
|  | |  | Obs. | 0,936 | 0,818 | 0,519 | 0,274 | 0,978 | 0,830 | 0,354 | 0,110 | 0,978 | 0,830 | 0,354 | 0,110 |
| **100** | | **1** | **Theo.** | **0,994** | **0,941** | **0,604** | **0,228** | **0,994** | **0,941** | **0,603** | **0,225** | **0,994** | **0,941** | **0,603** | **0,225** |
|  | |  | Obs. | 0,989 | 0,940 | 0,608 | 0,206 | 0,989 | 0,940 | 0,607 | 0,203 | 0,989 | 0,940 | 0,607 | 0,203 |
| **100** | | **1.5** | **Theo.** | **0,998** | **0,971** | **0,651** | **0,188** | **0,996** | **0,971** | **0,739** | **0,307** | **0,996** | **0,971** | **0,739** | **0,307** |
|  | |  | Obs. | 0,996 | 0,970 | 0,658 | 0,169 | 0,992 | 0,966 | 0,746 | 0,293 | 0,992 | 0,966 | 0,746 | 0,293 |
| **100** | | **2** | **Theo.** | **0,999** | **0,982** | **0,683** | **0,162** | **0,997** | **0,982** | **0,820** | **0,382** | **0,997** | **0,982** | **0,820** | **0,382** |
|  | |  | Obs. | 0,998 | 0,981 | 0,692 | 0,141 | 0,993 | 0,977 | 0,825 | 0,373 | 0,993 | 0,977 | 0,825 | 0,373 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.4b  *Comparison between observed and expected power, when nominal alpha risk = 5%, three groups are compared and samples are extracted from normal right skewed distributions.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,192** | **0,214** | **0,220** | **0,232** | **0,463** | **0,209** | **0,095** | **0,062** | **0,282** | **0,209** | **0,124** | **0,086** |
|  | |  | Obs. | 0,222 | 0,215 | 0,194 | 0,214 | 0,498 | 0,180 | 0,067 | 0,054 | 0,302 | 0,187 | 0,085 | 0,071 |
| **20** | | **1** | **Theo.** | **0,452** | **0,338** | **0,196** | **0,124** | **0,593** | **0,329** | **0,136** | **0,073** | **0,443** | **0,336** | **0,188** | **0,111** |
|  | |  | Obs. | 0,470 | 0,339 | 0,167 | 0,100 | 0,630 | 0,342 | 0,112 | 0,058 | 0,461 | 0,337 | 0,157 | 0,087 |
| **20** | | **1.5** | **Theo.** | **0,627** | **0,424** | **0,181** | **0,076** | **0,647** | **0,413** | **0,176** | **0,083** | **0,532** | **0,421** | **0,245** | **0,132** |
|  | |  | Obs. | 0,628 | 0,430 | 0,151 | 0,054 | 0,675 | 0,446 | 0,162 | 0,068 | 0,540 | 0,430 | 0,224 | 0,107 |
| **20** | | **2** | **Theo.** | **0,732** | **0,487** | **0,169** | **0,050** | **0,677** | **0,472** | **0,213** | **0,093** | **0,586** | **0,481** | **0,292** | **0,151** |
|  | |  | Obs. | 0,725 | 0,495 | 0,139 | 0,032 | 0,699 | 0,512 | 0,209 | 0,079 | 0,587 | 0,489 | 0,279 | 0,127 |
| **30** | | **0.5** | **Theo.** | **0,320** | **0,310** | **0,267** | **0,242** | **0,657** | **0,303** | **0,118** | **0,068** | **0,440** | **0,305** | **0,162** | **0,099** |
|  | |  | Obs. | 0,343 | 0,308 | 0,243 | 0,222 | 0,687 | 0,282 | 0,087 | 0,055 | 0,451 | 0,289 | 0,122 | 0,078 |
| **30** | | **1** | **Theo.** | **0,659** | **0,490** | **0,267** | **0,143** | **0,793** | **0,482** | **0,187** | **0,085** | **0,654** | **0,489** | **0,261** | **0,133** |
|  | |  | Obs. | 0,655 | 0,491 | 0,242 | 0,118 | 0,804 | 0,501 | 0,164 | 0,067 | 0,649 | 0,490 | 0,235 | 0,108 |
| **30** | | **1.5** | **Theo.** | **0,817** | **0,602** | **0,263** | **0,098** | **0,841** | **0,594** | **0,251** | **0,102** | **0,748** | **0,600** | **0,341** | **0,165** |
|  | |  | Obs. | 0,801 | 0,607 | 0,240 | 0,073 | 0,840 | 0,620 | 0,241 | 0,085 | 0,732 | 0,601 | 0,328 | 0,140 |
| **30** | | **2** | **Theo.** | **0,889** | **0,675** | **0,260** | **0,070** | **0,864** | **0,666** | **0,309** | **0,119** | **0,797** | **0,671** | **0,410** | **0,194** |
|  | |  | Obs. | 0,874 | 0,683 | 0,236 | 0,048 | 0,858 | 0,690 | 0,310 | 0,104 | 0,776 | 0,670 | 0,402 | 0,172 |
| **40** | | **0.5** | **Theo.** | **0,463** | **0,404** | **0,315** | **0,256** | **0,796** | **0,395** | **0,145** | **0,074** | **0,591** | **0,400** | **0,202** | **0,112** |
|  | |  | Obs. | 0,472 | 0,401 | 0,292 | 0,235 | 0,818 | 0,386 | 0,111 | 0,058 | 0,592 | 0,390 | 0,163 | 0,087 |
| **40** | | **1** | **Theo.** | **0,808** | **0,621** | **0,334** | **0,164** | **0,904** | **0,615** | **0,239** | **0,097** | **0,805** | **0,621** | **0,330** | **0,156** |
|  | |  | Obs. | 0,792 | 0,622 | 0,315 | 0,139 | 0,904 | 0,637 | 0,218 | 0,078 | 0,789 | 0,622 | 0,309 | 0,131 |
| **40** | | **1.5** | **Theo.** | **0,919** | **0,740** | **0,346** | **0,119** | **0,935** | **0,734** | **0,326** | **0,121** | **0,878** | **0,739** | **0,432** | **0,198** |
|  | |  | Obs. | 0,903 | 0,743 | 0,329 | 0,095 | 0,928 | 0,754 | 0,321 | 0,104 | 0,857 | 0,735 | 0,426 | 0,175 |
| **40** | | **2** | **Theo.** | **0,958** | **0,808** | **0,353** | **0,091** | **0,949** | **0,802** | **0,404** | **0,145** | **0,912** | **0,805** | **0,516** | **0,236** |
|  | |  | Obs. | 0,947 | 0,812 | 0,337 | 0,068 | 0,940 | 0,815 | 0,409 | 0,130 | 0,891 | 0,798 | 0,513 | 0,216 |
| **50** | | **0.5** | **Theo.** | **0,595** | **0,492** | **0,358** | **0,270** | **0,884** | **0,484** | **0,171** | **0,080** | **0,716** | **0,489** | **0,240** | **0,124** |
|  | |  | Obs. | 0,594 | 0,489 | 0,340 | 0,251 | 0,899 | 0,487 | 0,137 | 0,062 | 0,711 | 0,485 | 0,205 | 0,098 |
| **50** | | **1** | **Theo.** | **0,900** | **0,727** | **0,400** | **0,186** | **0,959** | **0,722** | **0,293** | **0,110** | **0,899** | **0,726** | **0,396** | **0,179** |
|  | |  | Obs. | 0,884 | 0,728 | 0,386 | 0,161 | 0,956 | 0,745 | 0,275 | 0,091 | 0,882 | 0,728 | 0,381 | 0,154 |
| **50** | | **1.5** | **Theo.** | **0,967** | **0,836** | **0,426** | **0,142** | **0,976** | **0,832** | **0,401** | **0,141** | **0,946** | **0,835** | **0,517** | **0,229** |
|  | |  | Obs. | 0,957 | 0,839 | 0,416 | 0,117 | 0,970 | 0,847 | 0,400 | 0,123 | 0,930 | 0,832 | 0,513 | 0,208 |
| **50** | | **2** | **Theo.** | **0,986** | **0,891** | **0,443** | **0,112** | **0,982** | **0,888** | **0,493** | **0,171** | **0,965** | **0,890** | **0,608** | **0,276** |
|  | |  | Obs. | 0,979 | 0,893 | 0,436 | 0,090 | 0,976 | 0,895 | 0,503 | 0,158 | 0,950 | 0,882 | 0,610 | 0,260 |
| **100** | | **0.5** | **Theo.** | **0,948** | **0,810** | **0,550** | **0,341** | **0,996** | **0,804** | **0,310** | **0,111** | **0,975** | **0,809** | **0,421** | **0,182** |
|  | |  | Obs. | 0,937 | 0,810 | 0,546 | 0,325 | 0,997 | 0,834 | 0,285 | 0,090 | 0,971 | 0,819 | 0,405 | 0,156 |
| **100** | | **1** | **Theo.** | **0,998** | **0,962** | **0,900** | **0,291** | **1,000** | **0,961** | **0,600** | **0,178** | **0,998** | **0,962** | **0,900** | **0,287** |
|  | |  | Obs. | 0,997 | 0,963 | 0,674 | 0,274 | 1,000 | 0,970 | 0,547 | 0,158 | 0,997 | 0,963 | 0,672 | 0,269 |
| **100** | | **1.5** | **Theo.** | **1,000** | **0,989** | **0,736** | **0,262** | **1,000** | **0,989** | **0,705** | **0,247** | **1,000** | **0,989** | **0,805** | **0,381** |
|  | |  | Obs. | 1,000 | 0,990 | 0,748 | 0,243 | 1,000 | 0,991 | 0,721 | 0,231 | 0,999 | 0,989 | 0,815 | 0,370 |
| **100** | | **2** | **Theo.** | **1,000** | **0,996** | **0,783** | **0,238** | **1,000** | **0,996** | **0,811** | **0,312** | **1,000** | **0,996** | **0,884** | **0,464** |
|  | |  | Obs. | 1,000 | 0,996 | 0,796 | 0,219 | 1,000 | 0,996 | 0,826 | 0,303 | 1,000 | 0,995 | 0,891 | 0,458 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.5a  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared, one sample is extracted from normal right skewed distribution, and one sample is extracted from a left skewed distribution.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,265** | **0,239** | **0,206** | **0,192** | **0,419** | **0,232** | **0,107** | **0,065** | **0,419** | **0,232** | **0,107** | **0,065** |
|  | |  | Obs. | 0,311 | 0,278 | 0,246 | 0,224 | 0,443 | 0,280 | 0,161 | 0,106 | 0,443 | 0,280 | 0,161 | 0,106 |
| **20** | | **1** | **Theo.** | **0,498** | **0,338** | **0,168** | **0,089** | **0,489** | **0,337** | **0,162** | **0,082** | **0,489** | **0,337** | **0,162** | **0,082** |
|  | |  | Obs. | 0,509 | 0,367 | 0,207 | 0,123 | 0,501 | 0,366 | 0,202 | 0,116 | 0,501 | 0,366 | 0,202 | 0,116 |
| **20** | | **1.5** | **Theo.** | **0,618** | **0,397** | **0,146** | **0,050** | **0,513** | **0,394** | **0,207** | **0,097** | **0,513** | **0,394** | **0,207** | **0,097** |
|  | |  | Obs. | 0,614 | 0,417 | 0,183 | 0,079 | 0,524 | 0,415 | 0,241 | 0,129 | 0,524 | 0,415 | 0,241 | 0,129 |
| **20** | | **2** | **Theo.** | **0,686** | **0,435** | **0,131** | **0,031** | **0,525** | **0,428** | **0,245** | **0,113** | **0,525** | **0,428** | **0,245** | **0,113** |
|  | |  | Obs. | 0,676 | 0,451 | 0,167 | 0,053 | 0,535 | 0,448 | 0,274 | 0,141 | 0,535 | 0,448 | 0,274 | 0,141 |
| **30** | | **0.5** | **Theo.** | **0,414** | **0,339** | **0,250** | **0,201** | **0,588** | **0,332** | **0,140** | **0,073** | **0,588** | **0,332** | **0,140** | **0,073** |
|  | |  | Obs. | 0,436 | 0,366 | 0,285 | 0,231 | 0,586 | 0,365 | 0,189 | 0,111 | 0,586 | 0,365 | 0,189 | 0,111 |
| **30** | | **1** | **Theo.** | **0,673** | **0,478** | **0,227** | **0,105** | **0,667** | **0,478** | **0,223** | **0,099** | **0,667** | **0,478** | **0,223** | **0,099** |
|  | |  | Obs. | 0,661 | 0,488 | 0,262 | 0,136 | 0,656 | 0,487 | 0,258 | 0,131 | 0,656 | 0,487 | 0,258 | 0,131 |
| **30** | | **1.5** | **Theo.** | **0,778** | **0,553** | **0,214** | **0,064** | **0,695** | **0,551** | **0,291** | **0,124** | **0,695** | **0,551** | **0,291** | **0,124** |
|  | |  | Obs. | 0,759 | 0,554 | 0,245 | 0,091 | 0,680 | 0,552 | 0,316 | 0,151 | 0,680 | 0,552 | 0,316 | 0,151 |
| **30** | | **2** | **Theo.** | **0,832** | **0,600** | **0,203** | **0,043** | **0,710** | **0,594** | **0,347** | **0,147** | **0,710** | **0,594** | **0,347** | **0,147** |
|  | |  | Obs. | 0,811 | 0,598 | 0,232 | 0,065 | 0,691 | 0,592 | 0,365 | 0,172 | 0,691 | 0,592 | 0,365 | 0,172 |
| **40** | | **0.5** | **Theo.** | **0,553** | **0,435** | **0,294** | **0,214** | **0,718** | **0,429** | **0,173** | **0,082** | **0,718** | **0,429** | **0,173** | **0,082** |
|  | |  | Obs. | 0,554 | 0,450 | 0,323 | 0,242 | 0,702 | 0,447 | 0,218 | 0,118 | 0,702 | 0,447 | 0,218 | 0,118 |
| **40** | | **1** | **Theo.** | **0,797** | **0,599** | **0,288** | **0,121** | **0,794** | **0,598** | **0,285** | **0,116** | **0,794** | **0,598** | **0,285** | **0,116** |
|  | |  | Obs. | 0,774 | 0,596 | 0,316 | 0,151 | 0,771 | 0,595 | 0,312 | 0,147 | 0,771 | 0,595 | 0,312 | 0,147 |
| **40** | | **1.5** | **Theo.** | **0,879** | **0,679** | **0,284** | **0,080** | **0,820** | **0,677** | **0,373** | **0,149** | **0,820** | **0,677** | **0,373** | **0,149** |
|  | |  | Obs. | 0,855 | 0,670 | 0,308 | 0,106 | 0,794 | 0,668 | 0,389 | 0,175 | 0,794 | 0,668 | 0,389 | 0,175 |
| **40** | | **2** | **Theo.** | **0,914** | **0,726** | **0,278** | **0,056** | **0,833** | **0,722** | **0,440** | **0,181** | **0,833** | **0,722** | **0,440** | **0,181** |
|  | |  | Obs. | 0,894 | 0,714 | 0,302 | 0,079 | 0,805 | 0,708 | 0,451 | 0,204 | 0,805 | 0,708 | 0,451 | 0,204 |
| **50** | | **0.5** | **Theo.** | **0,670** | **0,522** | **0,337** | **0,226** | **0,813** | **0,516** | **0,208** | **0,091** | **0,813** | **0,516** | **0,208** | **0,091** |
|  | |  | Obs. | 0,657 | 0,528 | 0,361 | 0,253 | 0,790 | 0,523 | 0,248 | 0,126 | 0,790 | 0,523 | 0,248 | 0,126 |
| **50** | | **1** | **Theo.** | **0,879** | **0,697** | **0,348** | **0,139** | **0,877** | **0,697** | **0,345** | **0,135** | **0,877** | **0,697** | **0,345** | **0,135** |
|  | |  | Obs. | 0,854 | 0,685 | 0,368 | 0,167 | 0,852 | 0,685 | 0,365 | 0,163 | 0,852 | 0,685 | 0,365 | 0,163 |
| **50** | | **1.5** | **Theo.** | **0,935** | **0,776** | **0,353** | **0,097** | **0,897** | **0,775** | **0,449** | **0,176** | **0,897** | **0,775** | **0,449** | **0,176** |
|  | |  | Obs. | 0,915 | 0,760 | 0,371 | 0,122 | 0,871 | 0,758 | 0,459 | 0,200 | 0,871 | 0,758 | 0,459 | 0,200 |
| **50** | | **2** | **Theo.** | **0,957** | **0,818** | **0,356** | **0,071** | **0,906** | **0,815** | **0,527** | **0,215** | **0,906** | **0,815** | **0,527** | **0,215** |
|  | |  | Obs. | 0,942 | 0,803 | 0,371 | 0,093 | 0,881 | 0,797 | 0,530 | 0,236 | 0,881 | 0,797 | 0,530 | 0,236 |
| **100** | | **0.5** | **Theo.** | **0,950** | **0,818** | **0,525** | **0,294** | **0,982** | **0,815** | **0,375** | **0,137** | **0,982** | **0,815** | **0,375** | **0,137** |
|  | |  | Obs. | 0,932 | 0,802 | 0,530 | 0,313 | 0,972 | 0,796 | 0,395 | 0,166 | 0,972 | 0,796 | 0,395 | 0,166 |
| **100** | | **1** | **Theo.** | **0,994** | **0,941** | **0,604** | **0,228** | **0,994** | **0,941** | **0,603** | **0,225** | **0,994** | **0,941** | **0,603** | **0,225** |
|  | |  | Obs. | 0,988 | 0,926 | 0,602 | 0,248 | 0,988 | 0,926 | 0,601 | 0,246 | 0,988 | 0,926 | 0,601 | 0,246 |
| **100** | | **1.5** | **Theo.** | **0,998** | **0,971** | **0,651** | **0,188** | **0,996** | **0,971** | **0,739** | **0,307** | **0,996** | **0,971** | **0,739** | **0,307** |
|  | |  | Obs. | 0,996 | 0,962 | 0,645 | 0,211 | 0,992 | 0,961 | 0,729 | 0,324 | 0,992 | 0,961 | 0,729 | 0,324 |
| **100** | | **2** | **Theo.** | **0,999** | **0,982** | **0,683** | **0,162** | **0,997** | **0,982** | **0,820** | **0,382** | **0,997** | **0,982** | **0,820** | **0,382** |
|  | |  | Obs. | 0,998 | 0,976 | 0,674 | 0,182 | 0,993 | 0,974 | 0,808 | 0,392 | 0,993 | 0,974 | 0,808 | 0,392 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.5b  *Comparison between observed and expected power, when nominal alpha risk = 5%, three groups are compared, two samples are extracted from normal right skewed distribution, and one is extracted from a left skewed distribution.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,192** | **0,214** | **0,220** | **0,232** | **0,463** | **0,209** | **0,095** | **0,062** | **0,282** | **0,209** | **0,124** | **0,086** |
|  | |  | Obs. | 0,229 | 0,245 | 0,255 | 0,261 | 0,513 | 0,284 | 0,154 | 0,102 | 0,317 | 0,251 | 0,176 | 0,128 |
| **20** | | **1** | **Theo.** | **0,452** | **0,338** | **0,196** | **0,124** | **0,593** | **0,329** | **0,136** | **0,073** | **0,443** | **0,336** | **0,188** | **0,111** |
|  | |  | Obs. | 0,472 | 0,363 | 0,234 | 0,157 | 0,624 | 0,386 | 0,191 | 0,107 | 0,464 | 0,360 | 0,228 | 0,145 |
| **20** | | **1.5** | **Theo.** | **0,627** | **0,424** | **0,181** | **0,076** | **0,647** | **0,413** | **0,176** | **0,083** | **0,532** | **0,421** | **0,245** | **0,132** |
|  | |  | Obs. | 0,625 | 0,441 | 0,217 | 0,108 | 0,671 | 0,461 | 0,227 | 0,116 | 0,541 | 0,438 | 0,274 | 0,162 |
| **20** | | **2** | **Theo.** | **0,732** | **0,487** | **0,169** | **0,050** | **0,677** | **0,472** | **0,213** | **0,093** | **0,586** | **0,481** | **0,292** | **0,151** |
|  | |  | Obs. | 0,720 | 0,497 | 0,204 | 0,078 | 0,696 | 0,517 | 0,264 | 0,126 | 0,587 | 0,492 | 0,316 | 0,179 |
| **30** | | **0.5** | **Theo.** | **0,320** | **0,310** | **0,267** | **0,242** | **0,657** | **0,303** | **0,118** | **0,068** | **0,440** | **0,305** | **0,162** | **0,099** |
|  | |  | Obs. | 0,350 | 0,335 | 0,297 | 0,268 | 0,669 | 0,361 | 0,174 | 0,104 | 0,459 | 0,337 | 0,208 | 0,137 |
| **30** | | **1** | **Theo.** | **0,659** | **0,490** | **0,267** | **0,143** | **0,793** | **0,482** | **0,187** | **0,085** | **0,654** | **0,489** | **0,261** | **0,133** |
|  | |  | Obs. | 0,652 | 0,500 | 0,295 | 0,174 | 0,790 | 0,515 | 0,235 | 0,118 | 0,648 | 0,499 | 0,290 | 0,165 |
| **30** | | **1.5** | **Theo.** | **0,817** | **0,602** | **0,263** | **0,098** | **0,841** | **0,594** | **0,251** | **0,102** | **0,748** | **0,600** | **0,341** | **0,165** |
|  | |  | Obs. | 0,796 | 0,600 | 0,292 | 0,126 | 0,833 | 0,614 | 0,295 | 0,133 | 0,731 | 0,598 | 0,362 | 0,192 |
| **30** | | **2** | **Theo.** | **0,889** | **0,675** | **0,260** | **0,070** | **0,864** | **0,666** | **0,309** | **0,119** | **0,797** | **0,671** | **0,410** | **0,194** |
|  | |  | Obs. | 0,869 | 0,668 | 0,287 | 0,097 | 0,854 | 0,680 | 0,349 | 0,151 | 0,775 | 0,665 | 0,423 | 0,218 |
| **40** | | **0.5** | **Theo.** | **0,463** | **0,404** | **0,315** | **0,256** | **0,796** | **0,395** | **0,145** | **0,074** | **0,591** | **0,400** | **0,202** | **0,112** |
|  | |  | Obs. | 0,477 | 0,422 | 0,339 | 0,280 | 0,785 | 0,438 | 0,197 | 0,108 | 0,591 | 0,421 | 0,242 | 0,147 |
| **40** | | **1** | **Theo.** | **0,808** | **0,621** | **0,334** | **0,164** | **0,904** | **0,615** | **0,239** | **0,097** | **0,805** | **0,621** | **0,330** | **0,156** |
|  | |  | Obs. | 0,787 | 0,618 | 0,357 | 0,192 | 0,891 | 0,627 | 0,282 | 0,128 | 0,784 | 0,617 | 0,353 | 0,185 |
| **40** | | **1.5** | **Theo.** | **0,919** | **0,740** | **0,346** | **0,119** | **0,935** | **0,734** | **0,326** | **0,121** | **0,878** | **0,739** | **0,432** | **0,198** |
|  | |  | Obs. | 0,898 | 0,726 | 0,366 | 0,147 | 0,923 | 0,735 | 0,362 | 0,151 | 0,855 | 0,725 | 0,445 | 0,222 |
| **40** | | **2** | **Theo.** | **0,958** | **0,808** | **0,353** | **0,091** | **0,949** | **0,802** | **0,404** | **0,145** | **0,912** | **0,805** | **0,516** | **0,236** |
|  | |  | Obs. | 0,943 | 0,791 | 0,372 | 0,117 | 0,936 | 0,799 | 0,434 | 0,176 | 0,888 | 0,789 | 0,520 | 0,257 |
| **50** | | **0.5** | **Theo.** | **0,595** | **0,492** | **0,358** | **0,270** | **0,884** | **0,484** | **0,171** | **0,080** | **0,716** | **0,489** | **0,240** | **0,124** |
|  | |  | Obs. | 0,596 | 0,503 | 0,380 | 0,293 | 0,868 | 0,511 | 0,221 | 0,113 | 0,705 | 0,500 | 0,276 | 0,158 |
| **50** | | **1** | **Theo.** | **0,900** | **0,727** | **0,400** | **0,186** | **0,959** | **0,722** | **0,293** | **0,110** | **0,899** | **0,726** | **0,396** | **0,179** |
|  | |  | Obs. | 0,878 | 0,714 | 0,416 | 0,211 | 0,947 | 0,720 | 0,330 | 0,140 | 0,877 | 0,714 | 0,412 | 0,205 |
| **50** | | **1.5** | **Theo.** | **0,967** | **0,836** | **0,426** | **0,142** | **0,976** | **0,832** | **0,401** | **0,141** | **0,946** | **0,835** | **0,517** | **0,229** |
|  | |  | Obs. | 0,953 | 0,819 | 0,439 | 0,168 | 0,966 | 0,824 | 0,429 | 0,170 | 0,928 | 0,819 | 0,521 | 0,251 |
| **50** | | **2** | **Theo.** | **0,986** | **0,891** | **0,443** | **0,112** | **0,982** | **0,888** | **0,493** | **0,171** | **0,965** | **0,890** | **0,608** | **0,276** |
|  | |  | Obs. | 0,978 | 0,874 | 0,453 | 0,138 | 0,974 | 0,879 | 0,513 | 0,201 | 0,949 | 0,873 | 0,605 | 0,295 |
| **100** | | **0.5** | **Theo.** | **0,948** | **0,810** | **0,550** | **0,341** | **0,996** | **0,804** | **0,310** | **0,111** | **0,975** | **0,809** | **0,421** | **0,182** |
|  | |  | Obs. | 0,932 | 0,800 | 0,555 | 0,357 | 0,992 | 0,789 | 0,344 | 0,141 | 0,964 | 0,793 | 0,437 | 0,210 |
| **100** | | **1** | **Theo.** | **0,998** | **0,962** | **0,900** | **0,291** | **1,000** | **0,961** | **0,600** | **0,178** | **0,998** | **0,962** | **0,900** | **0,287** |
|  | |  | Obs. | 0,996 | 0,951 | 0,662 | 0,310 | 0,999 | 0,950 | 0,552 | 0,204 | 0,996 | 0,951 | 0,660 | 0,306 |
| **100** | | **1.5** | **Theo.** | **1,000** | **0,989** | **0,736** | **0,262** | **1,000** | **0,989** | **0,705** | **0,247** | **1,000** | **0,989** | **0,805** | **0,381** |
|  | |  | Obs. | 1,000 | 0,984 | 0,726 | 0,280 | 1,000 | 0,985 | 0,703 | 0,270 | 0,999 | 0,984 | 0,794 | 0,393 |
| **100** | | **2** | **Theo.** | **1,000** | **0,996** | **0,783** | **0,238** | **1,000** | **0,996** | **0,811** | **0,312** | **1,000** | **0,996** | **0,884** | **0,464** |
|  | |  | Obs. | 1,000 | 0,994 | 0,769 | 0,256 | 1,000 | 0,994 | 0,803 | 0,332 | 0,999 | 0,994 | 0,872 | 0,470 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.6a  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared, one sample is extracted from a chi-square distribution with 2 degrees of freedom, and one sample is extracted from a right skewed distribution.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,265** | **0,239** | **0,206** | **0,192** | **0,419** | **0,232** | **0,107** | **0,065** | **0,419** | **0,232** | **0,107** | **0,065** |
|  | |  | Obs. | 0,358 | 0,275 | 0,183 | 0,174 | 0,468 | 0,211 | 0,069 | 0,053 | 0,468 | 0,211 | 0,069 | 0,053 |
| **20** | | **1** | **Theo.** | **0,498** | **0,338** | **0,168** | **0,089** | **0,489** | **0,337** | **0,162** | **0,082** | **0,489** | **0,337** | **0,162** | **0,082** |
|  | |  | Obs. | 0,541 | 0,371 | 0,143 | 0,068 | 0,534 | 0,370 | 0,135 | 0,061 | 0,534 | 0,370 | 0,135 | 0,061 |
| **20** | | **1.5** | **Theo.** | **0,618** | **0,397** | **0,146** | **0,050** | **0,513** | **0,394** | **0,207** | **0,097** | **0,513** | **0,394** | **0,207** | **0,097** |
|  | |  | Obs. | 0,632 | 0,429 | 0,123 | 0,032 | 0,554 | 0,441 | 0,199 | 0,075 | 0,554 | 0,441 | 0,199 | 0,075 |
| **20** | | **2** | **Theo.** | **0,686** | **0,435** | **0,131** | **0,031** | **0,525** | **0,428** | **0,245** | **0,113** | **0,525** | **0,428** | **0,245** | **0,113** |
|  | |  | Obs. | 0,687 | 0,466 | 0,108 | 0,017 | 0,564 | 0,478 | 0,252 | 0,093 | 0,564 | 0,478 | 0,252 | 0,093 |
| **30** | | **0.5** | **Theo.** | **0,414** | **0,339** | **0,250** | **0,201** | **0,588** | **0,332** | **0,140** | **0,073** | **0,588** | **0,332** | **0,140** | **0,073** |
|  | |  | Obs. | 0,468 | 0,366 | 0,230 | 0,181 | 0,603 | 0,325 | 0,099 | 0,055 | 0,603 | 0,325 | 0,099 | 0,055 |
| **30** | | **1** | **Theo.** | **0,673** | **0,478** | **0,227** | **0,105** | **0,667** | **0,478** | **0,223** | **0,099** | **0,667** | **0,478** | **0,223** | **0,099** |
|  | |  | Obs. | 0,668 | 0,499 | 0,207 | 0,080 | 0,663 | 0,498 | 0,201 | 0,075 | 0,663 | 0,498 | 0,201 | 0,075 |
| **30** | | **1.5** | **Theo.** | **0,778** | **0,553** | **0,214** | **0,064** | **0,695** | **0,551** | **0,291** | **0,124** | **0,695** | **0,551** | **0,291** | **0,124** |
|  | |  | Obs. | 0,757 | 0,570 | 0,194 | 0,044 | 0,683 | 0,568 | 0,287 | 0,101 | 0,683 | 0,568 | 0,287 | 0,101 |
| **30** | | **2** | **Theo.** | **0,832** | **0,600** | **0,203** | **0,043** | **0,710** | **0,594** | **0,347** | **0,147** | **0,710** | **0,594** | **0,347** | **0,147** |
|  | |  | Obs. | 0,804 | 0,614 | 0,184 | 0,026 | 0,693 | 0,606 | 0,354 | 0,127 | 0,693 | 0,606 | 0,354 | 0,127 |
| **40** | | **0.5** | **Theo.** | **0,553** | **0,435** | **0,294** | **0,214** | **0,718** | **0,429** | **0,173** | **0,082** | **0,718** | **0,429** | **0,173** | **0,082** |
|  | |  | Obs. | 0,568 | 0,452 | 0,276 | 0,193 | 0,709 | 0,429 | 0,134 | 0,060 | 0,709 | 0,429 | 0,134 | 0,060 |
| **40** | | **1** | **Theo.** | **0,797** | **0,599** | **0,288** | **0,121** | **0,794** | **0,598** | **0,285** | **0,116** | **0,794** | **0,598** | **0,285** | **0,116** |
|  | |  | Obs. | 0,767 | 0,607 | 0,272 | 0,096 | 0,763 | 0,607 | 0,267 | 0,092 | 0,763 | 0,607 | 0,267 | 0,092 |
| **40** | | **1.5** | **Theo.** | **0,879** | **0,679** | **0,284** | **0,080** | **0,820** | **0,677** | **0,373** | **0,149** | **0,820** | **0,677** | **0,373** | **0,149** |
|  | |  | Obs. | 0,844 | 0,684 | 0,269 | 0,058 | 0,782 | 0,676 | 0,371 | 0,127 | 0,782 | 0,676 | 0,371 | 0,127 |
| **40** | | **2** | **Theo.** | **0,914** | **0,726** | **0,278** | **0,056** | **0,833** | **0,722** | **0,440** | **0,181** | **0,833** | **0,722** | **0,440** | **0,181** |
|  | |  | Obs. | 0,881 | 0,728 | 0,266 | 0,038 | 0,790 | 0,710 | 0,447 | 0,163 | 0,790 | 0,710 | 0,447 | 0,163 |
| **50** | | **0.5** | **Theo.** | **0,670** | **0,522** | **0,337** | **0,226** | **0,813** | **0,516** | **0,208** | **0,091** | **0,813** | **0,516** | **0,208** | **0,091** |
|  | |  | Obs. | 0,658 | 0,530 | 0,321 | 0,205 | 0,793 | 0,521 | 0,170 | 0,067 | 0,793 | 0,521 | 0,170 | 0,067 |
| **50** | | **1** | **Theo.** | **0,879** | **0,697** | **0,348** | **0,139** | **0,877** | **0,697** | **0,345** | **0,135** | **0,877** | **0,697** | **0,345** | **0,135** |
|  | |  | Obs. | 0,842 | 0,696 | 0,335 | 0,114 | 0,840 | 0,696 | 0,331 | 0,109 | 0,840 | 0,696 | 0,331 | 0,109 |
| **50** | | **1.5** | **Theo.** | **0,935** | **0,776** | **0,353** | **0,097** | **0,897** | **0,775** | **0,449** | **0,176** | **0,897** | **0,775** | **0,449** | **0,176** |
|  | |  | Obs. | 0,902 | 0,771 | 0,343 | 0,075 | 0,853 | 0,760 | 0,449 | 0,155 | 0,853 | 0,760 | 0,449 | 0,155 |
| **50** | | **2** | **Theo.** | **0,957** | **0,818** | **0,356** | **0,071** | **0,906** | **0,815** | **0,527** | **0,215** | **0,906** | **0,815** | **0,527** | **0,215** |
|  | |  | Obs. | 0,929 | 0,813 | 0,348 | 0,052 | 0,860 | 0,792 | 0,533 | 0,198 | 0,860 | 0,792 | 0,533 | 0,198 |
| **100** | | **0.5** | **Theo.** | **0,950** | **0,818** | **0,525** | **0,294** | **0,982** | **0,815** | **0,375** | **0,137** | **0,982** | **0,815** | **0,375** | **0,137** |
|  | |  | Obs. | 0,918 | 0,809 | 0,521 | 0,274 | 0,969 | 0,825 | 0,355 | 0,110 | 0,969 | 0,825 | 0,355 | 0,110 |
| **100** | | **1** | **Theo.** | **0,994** | **0,941** | **0,604** | **0,228** | **0,994** | **0,941** | **0,603** | **0,225** | **0,994** | **0,941** | **0,603** | **0,225** |
|  | |  | Obs. | 0,982 | 0,932 | 0,608 | 0,206 | 0,982 | 0,932 | 0,606 | 0,203 | 0,982 | 0,932 | 0,606 | 0,203 |
| **100** | | **1.5** | **Theo.** | **0,998** | **0,971** | **0,651** | **0,188** | **0,996** | **0,971** | **0,739** | **0,307** | **0,996** | **0,971** | **0,739** | **0,307** |
|  | |  | Obs. | 0,993 | 0,963 | 0,659 | 0,169 | 0,985 | 0,957 | 0,744 | 0,293 | 0,985 | 0,957 | 0,744 | 0,293 |
| **100** | | **2** | **Theo.** | **0,999** | **0,982** | **0,683** | **0,162** | **0,997** | **0,982** | **0,820** | **0,382** | **0,997** | **0,982** | **0,820** | **0,382** |
|  | |  | Obs. | 0,996 | 0,977 | 0,692 | 0,142 | 0,987 | 0,969 | 0,821 | 0,374 | 0,987 | 0,969 | 0,821 | 0,374 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.6b  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared, one sample is extracted from a chi-square distribution with 2 degrees of freedom, and one sample is extracted from a right skewed distribution.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,192** | **0,214** | **0,220** | **0,232** | **0,463** | **0,209** | **0,095** | **0,062** | **0,282** | **0,209** | **0,124** | **0,086** |
|  | |  | Obs. | 0,267 | 0,238 | 0,198 | 0,213 | 0,554 | 0,202 | 0,069 | 0,052 | 0,340 | 0,199 | 0,085 | 0,070 |
| **20** | | **1** | **Theo.** | **0,452** | **0,338** | **0,196** | **0,124** | **0,593** | **0,329** | **0,136** | **0,073** | **0,443** | **0,336** | **0,188** | **0,111** |
|  | |  | Obs. | 0,500 | 0,362 | 0,172 | 0,100 | 0,678 | 0,387 | 0,121 | 0,057 | 0,491 | 0,358 | 0,161 | 0,086 |
| **20** | | **1.5** | **Theo.** | **0,627** | **0,424** | **0,181** | **0,076** | **0,647** | **0,413** | **0,176** | **0,083** | **0,532** | **0,421** | **0,245** | **0,132** |
|  | |  | Obs. | 0,640 | 0,448 | 0,155 | 0,054 | 0,718 | 0,498 | 0,179 | 0,068 | 0,560 | 0,450 | 0,230 | 0,108 |
| **20** | | **2** | **Theo.** | **0,732** | **0,487** | **0,169** | **0,050** | **0,677** | **0,472** | **0,213** | **0,093** | **0,586** | **0,481** | **0,292** | **0,151** |
|  | |  | Obs. | 0,725 | 0,510 | 0,143 | 0,032 | 0,737 | 0,565 | 0,234 | 0,082 | 0,600 | 0,507 | 0,289 | 0,129 |
| **30** | | **0.5** | **Theo.** | **0,320** | **0,310** | **0,267** | **0,242** | **0,657** | **0,303** | **0,118** | **0,068** | **0,440** | **0,305** | **0,162** | **0,099** |
|  | |  | Obs. | 0,379 | 0,327 | 0,249 | 0,223 | 0,715 | 0,306 | 0,091 | 0,053 | 0,477 | 0,302 | 0,124 | 0,077 |
| **30** | | **1** | **Theo.** | **0,659** | **0,490** | **0,267** | **0,143** | **0,793** | **0,482** | **0,187** | **0,085** | **0,654** | **0,489** | **0,261** | **0,133** |
|  | |  | Obs. | 0,658 | 0,504 | 0,245 | 0,119 | 0,817 | 0,536 | 0,175 | 0,068 | 0,652 | 0,502 | 0,237 | 0,108 |
| **30** | | **1.5** | **Theo.** | **0,817** | **0,602** | **0,263** | **0,098** | **0,841** | **0,594** | **0,251** | **0,102** | **0,748** | **0,600** | **0,341** | **0,165** |
|  | |  | Obs. | 0,791 | 0,613 | 0,243 | 0,073 | 0,848 | 0,652 | 0,258 | 0,087 | 0,726 | 0,607 | 0,333 | 0,141 |
| **30** | | **2** | **Theo.** | **0,889** | **0,675** | **0,260** | **0,070** | **0,864** | **0,666** | **0,309** | **0,119** | **0,797** | **0,671** | **0,410** | **0,194** |
|  | |  | Obs. | 0,859 | 0,684 | 0,240 | 0,048 | 0,863 | 0,717 | 0,334 | 0,108 | 0,764 | 0,670 | 0,409 | 0,172 |
| **40** | | **0.5** | **Theo.** | **0,463** | **0,404** | **0,315** | **0,256** | **0,796** | **0,395** | **0,145** | **0,074** | **0,591** | **0,400** | **0,202** | **0,112** |
|  | |  | Obs. | 0,493 | 0,415 | 0,297 | 0,236 | 0,828 | 0,409 | 0,116 | 0,057 | 0,601 | 0,401 | 0,165 | 0,087 |
| **40** | | **1** | **Theo.** | **0,808** | **0,621** | **0,334** | **0,164** | **0,904** | **0,615** | **0,239** | **0,097** | **0,805** | **0,621** | **0,330** | **0,156** |
|  | |  | Obs. | 0,780 | 0,626 | 0,319 | 0,140 | 0,903 | 0,660 | 0,231 | 0,080 | 0,777 | 0,625 | 0,313 | 0,132 |
| **40** | | **1.5** | **Theo.** | **0,919** | **0,740** | **0,346** | **0,119** | **0,935** | **0,734** | **0,326** | **0,121** | **0,878** | **0,739** | **0,432** | **0,198** |
|  | |  | Obs. | 0,887 | 0,740 | 0,332 | 0,095 | 0,924 | 0,769 | 0,338 | 0,107 | 0,840 | 0,730 | 0,428 | 0,175 |
| **40** | | **2** | **Theo.** | **0,958** | **0,808** | **0,353** | **0,091** | **0,949** | **0,802** | **0,404** | **0,145** | **0,912** | **0,805** | **0,516** | **0,236** |
|  | |  | Obs. | 0,932 | 0,805 | 0,341 | 0,068 | 0,933 | 0,824 | 0,431 | 0,135 | 0,870 | 0,788 | 0,517 | 0,216 |
| **50** | | **0.5** | **Theo.** | **0,595** | **0,492** | **0,358** | **0,270** | **0,884** | **0,484** | **0,171** | **0,080** | **0,716** | **0,489** | **0,240** | **0,124** |
|  | |  | Obs. | 0,600 | 0,499 | 0,342 | 0,251 | 0,900 | 0,508 | 0,142 | 0,062 | 0,707 | 0,493 | 0,206 | 0,098 |
| **50** | | **1** | **Theo.** | **0,900** | **0,727** | **0,400** | **0,186** | **0,959** | **0,722** | **0,293** | **0,110** | **0,899** | **0,726** | **0,396** | **0,179** |
|  | |  | Obs. | 0,866 | 0,726 | 0,388 | 0,161 | 0,951 | 0,759 | 0,287 | 0,092 | 0,864 | 0,725 | 0,383 | 0,154 |
| **50** | | **1.5** | **Theo.** | **0,967** | **0,836** | **0,426** | **0,142** | **0,976** | **0,832** | **0,401** | **0,141** | **0,946** | **0,835** | **0,517** | **0,229** |
|  | |  | Obs. | 0,942 | 0,832 | 0,417 | 0,118 | 0,964 | 0,853 | 0,416 | 0,127 | 0,912 | 0,823 | 0,515 | 0,208 |
| **50** | | **2** | **Theo.** | **0,986** | **0,891** | **0,443** | **0,112** | **0,982** | **0,888** | **0,493** | **0,171** | **0,965** | **0,890** | **0,608** | **0,276** |
|  | |  | Obs. | 0,969 | 0,885 | 0,439 | 0,090 | 0,969 | 0,895 | 0,522 | 0,163 | 0,933 | 0,869 | 0,610 | 0,260 |
| **100** | | **0.5** | **Theo.** | **0,948** | **0,810** | **0,550** | **0,341** | **0,996** | **0,804** | **0,310** | **0,111** | **0,975** | **0,809** | **0,421** | **0,182** |
|  | |  | Obs. | 0,922 | 0,805 | 0,545 | 0,326 | 0,996 | 0,838 | 0,290 | 0,091 | 0,961 | 0,815 | 0,406 | 0,156 |
| **100** | | **1** | **Theo.** | **0,998** | **0,962** | **0,900** | **0,291** | **1,000** | **0,961** | **0,600** | **0,178** | **0,998** | **0,962** | **0,900** | **0,287** |
|  | |  | Obs. | 0,994 | 0,958 | 0,673 | 0,273 | 0,999 | 0,969 | 0,555 | 0,162 | 0,994 | 0,957 | 0,671 | 0,268 |
| **100** | | **1.5** | **Theo.** | **1,000** | **0,989** | **0,736** | **0,262** | **1,000** | **0,989** | **0,705** | **0,247** | **1,000** | **0,989** | **0,805** | **0,381** |
|  | |  | Obs. | 0,999 | 0,987 | 0,747 | 0,243 | 0,999 | 0,989 | 0,728 | 0,235 | 0,998 | 0,985 | 0,812 | 0,370 |
| **100** | | **2** | **Theo.** | **1,000** | **0,996** | **0,783** | **0,238** | **1,000** | **0,996** | **0,811** | **0,312** | **1,000** | **0,996** | **0,884** | **0,464** |
|  | |  | Obs. | 1,000 | 0,995 | 0,795 | 0,220 | 1,000 | 0,995 | 0,831 | 0,309 | 0,999 | 0,993 | 0,888 | 0,459 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.7a  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared, one sample is extracted from a chi-square distribution with 2 degrees of freedom, and one is extracted from a left skewed distribution.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,265** | **0,239** | **0,206** | **0,192** | **0,419** | **0,232** | **0,107** | **0,065** | **0,419** | **0,232** | **0,107** | **0,065** |
|  | |  | Obs. | 0,100 | 0,305 | 0,252 | 0,226 | 0,300 | 0,287 | 0,162 | 0,107 | 0,300 | 0,287 | 0,162 | 0,107 |
| **20** | | **1** | **Theo.** | **0,498** | **0,338** | **0,168** | **0,089** | **0,489** | **0,337** | **0,162** | **0,082** | **0,489** | **0,337** | **0,162** | **0,082** |
|  | |  | Obs. | 0,538 | 0,387 | 0,212 | 0,124 | 0,531 | 0,385 | 0,207 | 0,117 | 0,531 | 0,385 | 0,207 | 0,117 |
| **20** | | **1.5** | **Theo.** | **0,618** | **0,397** | **0,146** | **0,050** | **0,513** | **0,394** | **0,207** | **0,097** | **0,513** | **0,394** | **0,207** | **0,097** |
|  | |  | Obs. | 0,629 | 0,435 | 0,188 | 0,079 | 0,553 | 0,442 | 0,249 | 0,129 | 0,553 | 0,442 | 0,249 | 0,129 |
| **20** | | **2** | **Theo.** | **0,686** | **0,435** | **0,131** | **0,031** | **0,525** | **0,428** | **0,245** | **0,113** | **0,525** | **0,428** | **0,245** | **0,113** |
|  | |  | Obs. | 0,681 | 0,466 | 0,169 | 0,054 | 0,562 | 0,476 | 0,286 | 0,142 | 0,562 | 0,476 | 0,286 | 0,142 |
| **30** | | **0.5** | **Theo.** | **0,414** | **0,339** | **0,250** | **0,201** | **0,588** | **0,332** | **0,140** | **0,073** | **0,588** | **0,332** | **0,140** | **0,073** |
|  | |  | Obs. | 0,469 | 0,385 | 0,288 | 0,231 | 0,595 | 0,371 | 0,189 | 0,111 | 0,595 | 0,371 | 0,189 | 0,111 |
| **30** | | **1** | **Theo.** | **0,673** | **0,478** | **0,227** | **0,105** | **0,667** | **0,478** | **0,223** | **0,099** | **0,667** | **0,478** | **0,223** | **0,099** |
|  | |  | Obs. | 0,664 | 0,500 | 0,265 | 0,137 | 0,660 | 0,499 | 0,261 | 0,131 | 0,660 | 0,499 | 0,261 | 0,131 |
| **30** | | **1.5** | **Theo.** | **0,778** | **0,553** | **0,214** | **0,064** | **0,695** | **0,551** | **0,291** | **0,124** | **0,695** | **0,551** | **0,291** | **0,124** |
|  | |  | Obs. | 0,753 | 0,562 | 0,249 | 0,092 | 0,682 | 0,563 | 0,322 | 0,153 | 0,682 | 0,563 | 0,322 | 0,153 |
| **30** | | **2** | **Theo.** | **0,832** | **0,600** | **0,203** | **0,043** | **0,710** | **0,594** | **0,347** | **0,147** | **0,710** | **0,594** | **0,347** | **0,147** |
|  | |  | Obs. | 0,800 | 0,604 | 0,235 | 0,065 | 0,691 | 0,601 | 0,373 | 0,174 | 0,691 | 0,601 | 0,373 | 0,174 |
| **40** | | **0.5** | **Theo.** | **0,553** | **0,435** | **0,294** | **0,214** | **0,718** | **0,429** | **0,173** | **0,082** | **0,718** | **0,429** | **0,173** | **0,082** |
|  | |  | Obs. | 0,568 | 0,463 | 0,327 | 0,241 | 0,698 | 0,452 | 0,219 | 0,118 | 0,698 | 0,452 | 0,219 | 0,118 |
| **40** | | **1** | **Theo.** | **0,797** | **0,599** | **0,288** | **0,121** | **0,794** | **0,598** | **0,285** | **0,116** | **0,794** | **0,598** | **0,285** | **0,116** |
|  | |  | Obs. | 0,764 | 0,599 | 0,318 | 0,151 | 0,760 | 0,598 | 0,315 | 0,146 | 0,760 | 0,598 | 0,315 | 0,146 |
| **40** | | **1.5** | **Theo.** | **0,879** | **0,679** | **0,284** | **0,080** | **0,820** | **0,677** | **0,373** | **0,149** | **0,820** | **0,677** | **0,373** | **0,149** |
|  | |  | Obs. | 0,840 | 0,670 | 0,312 | 0,107 | 0,779 | 0,667 | 0,394 | 0,176 | 0,779 | 0,667 | 0,394 | 0,176 |
| **40** | | **2** | **Theo.** | **0,914** | **0,726** | **0,278** | **0,056** | **0,833** | **0,722** | **0,440** | **0,181** | **0,833** | **0,722** | **0,440** | **0,181** |
|  | |  | Obs. | 0,878 | 0,714 | 0,305 | 0,079 | 0,788 | 0,705 | 0,457 | 0,206 | 0,788 | 0,705 | 0,457 | 0,206 |
| **50** | | **0.5** | **Theo.** | **0,670** | **0,522** | **0,337** | **0,226** | **0,813** | **0,516** | **0,208** | **0,091** | **0,813** | **0,516** | **0,208** | **0,091** |
|  | |  | Obs. | 0,656 | 0,534 | 0,364 | 0,252 | 0,780 | 0,525 | 0,249 | 0,126 | 0,780 | 0,525 | 0,249 | 0,126 |
| **50** | | **1** | **Theo.** | **0,879** | **0,697** | **0,348** | **0,139** | **0,877** | **0,697** | **0,345** | **0,135** | **0,877** | **0,697** | **0,345** | **0,135** |
|  | |  | Obs. | 0,837 | 0,683 | 0,372 | 0,168 | 0,835 | 0,683 | 0,369 | 0,164 | 0,835 | 0,683 | 0,369 | 0,164 |
| **50** | | **1.5** | **Theo.** | **0,935** | **0,776** | **0,353** | **0,097** | **0,897** | **0,775** | **0,449** | **0,176** | **0,897** | **0,775** | **0,449** | **0,176** |
|  | |  | Obs. | 0,899 | 0,755 | 0,374 | 0,123 | 0,852 | 0,750 | 0,462 | 0,201 | 0,852 | 0,750 | 0,462 | 0,201 |
| **50** | | **2** | **Theo.** | **0,957** | **0,818** | **0,356** | **0,071** | **0,906** | **0,815** | **0,527** | **0,215** | **0,906** | **0,815** | **0,527** | **0,215** |
|  | |  | Obs. | 0,927 | 0,798 | 0,374 | 0,093 | 0,860 | 0,786 | 0,533 | 0,238 | 0,860 | 0,786 | 0,533 | 0,238 |
| **100** | | **0.5** | **Theo.** | **0,950** | **0,818** | **0,525** | **0,294** | **0,982** | **0,815** | **0,375** | **0,137** | **0,982** | **0,815** | **0,375** | **0,137** |
|  | |  | Obs. | 0,914 | 0,796 | 0,530 | 0,313 | 0,964 | 0,793 | 0,395 | 0,166 | 0,964 | 0,793 | 0,395 | 0,166 |
| **100** | | **1** | **Theo.** | **0,994** | **0,941** | **0,604** | **0,228** | **0,994** | **0,941** | **0,603** | **0,225** | **0,994** | **0,941** | **0,603** | **0,225** |
|  | |  | Obs. | 0,981 | 0,919 | 0,602 | 0,249 | 0,981 | 0,919 | 0,600 | 0,247 | 0,981 | 0,919 | 0,600 | 0,247 |
| **100** | | **1.5** | **Theo.** | **0,998** | **0,971** | **0,651** | **0,188** | **0,996** | **0,971** | **0,739** | **0,307** | **0,996** | **0,971** | **0,739** | **0,307** |
|  | |  | Obs. | 0,992 | 0,956 | 0,645 | 0,211 | 0,985 | 0,952 | 0,727 | 0,323 | 0,985 | 0,952 | 0,727 | 0,323 |
| **100** | | **2** | **Theo.** | **0,999** | **0,982** | **0,683** | **0,162** | **0,997** | **0,982** | **0,820** | **0,382** | **0,997** | **0,982** | **0,820** | **0,382** |
|  | |  | Obs. | 0,996 | 0,971 | 0,674 | 0,182 | 0,986 | 0,966 | 0,805 | 0,393 | 0,986 | 0,966 | 0,805 | 0,393 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.7b  *Comparison between observed and expected power, when nominal alpha risk = 5%, three groups are compared, two samples are extracted from a chi-square distribution with 2 degrees of freedom, and one is extracted from a left skewed distribution.* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,192** | **0,214** | **0,220** | **0,232** | **0,463** | **0,209** | **0,095** | **0,062** | **0,282** | **0,209** | **0,124** | **0,086** |
|  | |  | Obs. | 0,273 | 0,266 | 0,259 | 0,261 | 0,552 | 0,302 | 0,158 | 0,103 | 0,349 | 0,260 | 0,176 | 0,128 |
| **20** | | **1** | **Theo.** | **0,452** | **0,338** | **0,196** | **0,124** | **0,593** | **0,329** | **0,136** | **0,073** | **0,443** | **0,336** | **0,188** | **0,111** |
|  | |  | Obs. | 0,499 | 0,379 | 0,238 | 0,157 | 0,668 | 0,420 | 0,201 | 0,108 | 0,490 | 0,375 | 0,230 | 0,144 |
| **20** | | **1.5** | **Theo.** | **0,627** | **0,424** | **0,181** | **0,076** | **0,647** | **0,413** | **0,176** | **0,083** | **0,532** | **0,421** | **0,245** | **0,132** |
|  | |  | Obs. | 0,636 | 0,454 | 0,221 | 0,108 | 0,712 | 0,504 | 0,244 | 0,120 | 0,559 | 0,454 | 0,279 | 0,163 |
| **20** | | **2** | **Theo.** | **0,732** | **0,487** | **0,169** | **0,050** | **0,677** | **0,472** | **0,213** | **0,093** | **0,586** | **0,481** | **0,292** | **0,151** |
|  | |  | Obs. | 0,721 | 0,508 | 0,207 | 0,078 | 0,734 | 0,562 | 0,284 | 0,131 | 0,600 | 0,507 | 0,321 | 0,179 |
| **30** | | **0.5** | **Theo.** | **0,320** | **0,310** | **0,267** | **0,242** | **0,657** | **0,303** | **0,118** | **0,068** | **0,440** | **0,305** | **0,162** | **0,099** |
|  | |  | Obs. | 0,384 | 0,350 | 0,300 | 0,269 | 0,691 | 0,379 | 0,179 | 0,104 | 0,479 | 0,345 | 0,208 | 0,137 |
| **30** | | **1** | **Theo.** | **0,659** | **0,490** | **0,267** | **0,143** | **0,793** | **0,482** | **0,187** | **0,085** | **0,654** | **0,489** | **0,261** | **0,133** |
|  | |  | Obs. | 0,655 | 0,507 | 0,298 | 0,175 | 0,805 | 0,541 | 0,247 | 0,120 | 0,650 | 0,505 | 0,293 | 0,166 |
| **30** | | **1.5** | **Theo.** | **0,817** | **0,602** | **0,263** | **0,098** | **0,841** | **0,594** | **0,251** | **0,102** | **0,748** | **0,600** | **0,341** | **0,165** |
|  | |  | Obs. | 0,785 | 0,604 | 0,294 | 0,126 | 0,841 | 0,642 | 0,311 | 0,137 | 0,722 | 0,603 | 0,365 | 0,192 |
| **30** | | **2** | **Theo.** | **0,889** | **0,675** | **0,260** | **0,070** | **0,864** | **0,666** | **0,309** | **0,119** | **0,797** | **0,671** | **0,410** | **0,194** |
|  | |  | Obs. | 0,855 | 0,668 | 0,290 | 0,097 | 0,859 | 0,705 | 0,370 | 0,156 | 0,763 | 0,664 | 0,426 | 0,219 |
| **40** | | **0.5** | **Theo.** | **0,463** | **0,404** | **0,315** | **0,256** | **0,796** | **0,395** | **0,145** | **0,074** | **0,591** | **0,400** | **0,202** | **0,112** |
|  | |  | Obs. | 0,496 | 0,430 | 0,340 | 0,280 | 0,797 | 0,453 | 0,202 | 0,109 | 0,598 | 0,425 | 0,242 | 0,147 |
| **40** | | **1** | **Theo.** | **0,808** | **0,621** | **0,334** | **0,164** | **0,904** | **0,615** | **0,239** | **0,097** | **0,805** | **0,621** | **0,330** | **0,156** |
|  | |  | Obs. | 0,777 | 0,619 | 0,359 | 0,193 | 0,892 | 0,646 | 0,294 | 0,131 | 0,773 | 0,618 | 0,354 | 0,186 |
| **40** | | **1.5** | **Theo.** | **0,919** | **0,740** | **0,346** | **0,119** | **0,935** | **0,734** | **0,326** | **0,121** | **0,878** | **0,739** | **0,432** | **0,198** |
|  | |  | Obs. | 0,882 | 0,723 | 0,368 | 0,147 | 0,919 | 0,750 | 0,378 | 0,155 | 0,836 | 0,720 | 0,447 | 0,222 |
| **40** | | **2** | **Theo.** | **0,958** | **0,808** | **0,353** | **0,091** | **0,949** | **0,802** | **0,404** | **0,145** | **0,912** | **0,805** | **0,516** | **0,236** |
|  | |  | Obs. | 0,929 | 0,787 | 0,373 | 0,117 | 0,930 | 0,809 | 0,451 | 0,182 | 0,868 | 0,780 | 0,521 | 0,257 |
| **50** | | **0.5** | **Theo.** | **0,595** | **0,492** | **0,358** | **0,270** | **0,884** | **0,484** | **0,171** | **0,080** | **0,716** | **0,489** | **0,240** | **0,124** |
|  | |  | Obs. | 0,601 | 0,508 | 0,380 | 0,293 | 0,871 | 0,524 | 0,226 | 0,114 | 0,701 | 0,503 | 0,276 | 0,157 |
| **50** | | **1** | **Theo.** | **0,900** | **0,727** | **0,400** | **0,186** | **0,959** | **0,722** | **0,293** | **0,110** | **0,899** | **0,726** | **0,396** | **0,179** |
|  | |  | Obs. | 0,862 | 0,712 | 0,417 | 0,212 | 0,943 | 0,733 | 0,340 | 0,143 | 0,860 | 0,711 | 0,413 | 0,206 |
| **50** | | **1.5** | **Theo.** | **0,967** | **0,836** | **0,426** | **0,142** | **0,976** | **0,832** | **0,401** | **0,141** | **0,946** | **0,835** | **0,517** | **0,229** |
|  | |  | Obs. | 0,939 | 0,812 | 0,439 | 0,168 | 0,961 | 0,831 | 0,441 | 0,174 | 0,910 | 0,809 | 0,522 | 0,251 |
| **50** | | **2** | **Theo.** | **0,986** | **0,891** | **0,443** | **0,112** | **0,982** | **0,888** | **0,493** | **0,171** | **0,965** | **0,890** | **0,608** | **0,276** |
|  | |  | Obs. | 0,968 | 0,867 | 0,455 | 0,138 | 0,968 | 0,881 | 0,528 | 0,207 | 0,931 | 0,861 | 0,605 | 0,295 |
| **100** | | **0.5** | **Theo.** | **0,948** | **0,810** | **0,550** | **0,341** | **0,996** | **0,804** | **0,310** | **0,111** | **0,975** | **0,809** | **0,421** | **0,182** |
|  | |  | Obs. | 0,918 | 0,794 | 0,554 | 0,358 | 0,991 | 0,793 | 0,348 | 0,143 | 0,955 | 0,789 | 0,436 | 0,210 |
| **100** | | **1** | **Theo.** | **0,998** | **0,962** | **0,900** | **0,291** | **1,000** | **0,961** | **0,600** | **0,178** | **0,998** | **0,962** | **0,900** | **0,287** |
|  | |  | Obs. | 0,993 | 0,946 | 0,660 | 0,310 | 0,999 | 0,950 | 0,559 | 0,208 | 0,993 | 0,946 | 0,659 | 0,306 |
| **100** | | **1.5** | **Theo.** | **1,000** | **0,989** | **0,736** | **0,262** | **1,000** | **0,989** | **0,705** | **0,247** | **1,000** | **0,989** | **0,805** | **0,381** |
|  | |  | Obs. | 0,999 | 0,981 | 0,725 | 0,280 | 0,999 | 0,983 | 0,709 | 0,273 | 0,997 | 0,981 | 0,792 | 0,392 |
| **100** | | **2** | **Theo.** | **1,000** | **0,996** | **0,783** | **0,238** | **1,000** | **0,996** | **0,811** | **0,312** | **1,000** | **0,996** | **0,884** | **0,464** |
|  | |  | Obs. | 1,000 | 0,992 | 0,768 | 0,257 | 1,000 | 0,993 | 0,808 | 0,339 | 0,999 | 0,991 | 0,870 | 0,471 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.8a  *Comparison between observed and expected power, when nominal alpha risk = 5%, two groups are compared and samples are extracted from double exponential distributions, where standard deviation is ..* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,265** | **0,239** | **0,206** | **0,192** | **0,419** | **0,232** | **0,107** | **0,065** | **0,419** | **0,232** | **0,107** | **0,065** |
|  | |  | Obs. |  | 0,153 |  | 0,178 | 0,256 | 0,152 | 0,081 | 0,054 | 0,256 | 0,152 | 0,081 | 0,054 |
| **20** | | **1** | **Theo.** | **0,498** | **0,338** | **0,168** | **0,089** | **0,489** | **0,337** | **0,162** | **0,082** | **0,489** | **0,337** | **0,162** | **0,082** |
|  | |  | Obs. | 0,307 | 0,206 | 0,147 | 0,072 | 0,299 | 0,204 | 0,111 | 0,066 | 0,299 | 0,204 | 0,111 | 0,066 |
| **20** | | **1.5** | **Theo.** | **0,618** | **0,397** | **0,146** | **0,050** | **0,513** | **0,394** | **0,207** | **0,097** | **0,513** | **0,394** | **0,207** | **0,097** |
|  | |  | Obs. |  | 0,236 |  | 0,036 | 0,317 | 0,235 | 0,133 | 0,075 | 0,317 | 0,235 | 0,133 | 0,075 |
| **20** | | **2** | **Theo.** | **0,686** | **0,435** | **0,131** | **0,031** | **0,525** | **0,428** | **0,245** | **0,113** | **0,525** | **0,428** | **0,245** | **0,113** |
|  | |  | Obs. |  | 0,256 |  | 0,019 | 0,328 | 0,257 | 0,152 | 0,082 | 0,328 | 0,257 | 0,152 | 0,082 |
| **30** | | **0.5** | **Theo.** | **0,414** | **0,339** | **0,250** | **0,201** | **0,588** | **0,332** | **0,140** | **0,073** | **0,588** | **0,332** | **0,140** | **0,073** |
|  | |  | Obs. |  | 0,205 |  | 0,182 | 0,355 | 0,206 | 0,100 | 0,060 | 0,355 | 0,206 | 0,100 | 0,060 |
| **30** | | **1** | **Theo.** | **0,673** | **0,478** | **0,227** | **0,105** | **0,667** | **0,478** | **0,223** | **0,099** | **0,667** | **0,478** | **0,223** | **0,099** |
|  | |  | Obs. | 0,419 | 0,281 | 0,195 | 0,080 | 0,414 | 0,280 | 0,142 | 0,075 | 0,414 | 0,280 | 0,142 | 0,075 |
| **30** | | **1.5** | **Theo.** | **0,778** | **0,553** | **0,214** | **0,064** | **0,695** | **0,551** | **0,291** | **0,124** | **0,695** | **0,551** | **0,291** | **0,124** |
|  | |  | Obs. |  | 0,327 |  | 0,042 | 0,437 | 0,327 | 0,175 | 0,088 | 0,437 | 0,327 | 0,175 | 0,088 |
| **30** | | **2** | **Theo.** | **0,832** | **0,600** | **0,203** | **0,043** | **0,710** | **0,594** | **0,347** | **0,147** | **0,710** | **0,594** | **0,347** | **0,147** |
|  | |  | Obs. |  | 0,355 |  | 0,025 | 0,450 | 0,356 | 0,203 | 0,099 | 0,450 | 0,356 | 0,203 | 0,099 |
| **40** | | **0.5** | **Theo.** | **0,553** | **0,435** | **0,294** | **0,214** | **0,718** | **0,429** | **0,173** | **0,082** | **0,718** | **0,429** | **0,173** | **0,082** |
|  | |  | Obs. |  | 0,256 |  | 0,189 | 0,448 | 0,258 | 0,118 | 0,066 | 0,448 | 0,258 | 0,118 | 0,066 |
| **40** | | **1** | **Theo.** | **0,797** | **0,599** | **0,288** | **0,121** | **0,794** | **0,598** | **0,285** | **0,116** | **0,794** | **0,598** | **0,285** | **0,116** |
|  | |  | Obs. | 0,521 | 0,355 | 0,242 | 0,089 | 0,517 | 0,355 | 0,173 | 0,085 | 0,517 | 0,355 | 0,173 | 0,085 |
| **40** | | **1.5** | **Theo.** | **0,879** | **0,679** | **0,284** | **0,080** | **0,820** | **0,677** | **0,373** | **0,149** | **0,820** | **0,677** | **0,373** | **0,149** |
|  | |  | Obs. |  | 0,412 |  | 0,049 | 0,544 | 0,412 | 0,217 | 0,101 | 0,544 | 0,412 | 0,217 | 0,101 |
| **40** | | **2** | **Theo.** | **0,914** | **0,726** | **0,278** | **0,056** | **0,833** | **0,722** | **0,440** | **0,181** | **0,833** | **0,722** | **0,440** | **0,181** |
|  | |  | Obs. |  | 0,449 |  | 0,030 | 0,559 | 0,449 | 0,255 | 0,116 | 0,559 | 0,449 | 0,255 | 0,116 |
| **50** | | **0.5** | **Theo.** | **0,670** | **0,522** | **0,337** | **0,226** | **0,813** | **0,516** | **0,208** | **0,091** | **0,813** | **0,516** | **0,208** | **0,091** |
|  | |  | Obs. | 0,366 | 0,306 | 0,235 | 0,195 | 0,532 | 0,307 | 0,136 | 0,072 | 0,532 | 0,307 | 0,136 | 0,072 |
| **50** | | **1** | **Theo.** | **0,879** | **0,697** | **0,348** | **0,139** | **0,877** | **0,697** | **0,345** | **0,135** | **0,877** | **0,697** | **0,345** | **0,135** |
|  | |  | Obs. | 0,610 | 0,426 | 0,207 | 0,097 | 0,607 | 0,426 | 0,204 | 0,094 | 0,607 | 0,426 | 0,204 | 0,094 |
| **50** | | **1.5** | **Theo.** | **0,935** | **0,776** | **0,353** | **0,097** | **0,897** | **0,775** | **0,449** | **0,176** | **0,897** | **0,775** | **0,449** | **0,176** |
|  | |  | Obs. | 0,719 | 0,491 | 0,188 | 0,057 | 0,636 | 0,491 | 0,260 | 0,114 | 0,636 | 0,491 | 0,260 | 0,114 |
| **50** | | **2** | **Theo.** | **0,957** | **0,818** | **0,356** | **0,071** | **0,906** | **0,815** | **0,527** | **0,215** | **0,906** | **0,815** | **0,527** | **0,215** |
|  | |  | Obs. | 0,774 | 0,532 | 0,174 | 0,037 | 0,649 | 0,533 | 0,305 | 0,134 | 0,649 | 0,533 | 0,305 | 0,134 |
| **100** | | **0.5** | **Theo.** | **0,950** | **0,818** | **0,525** | **0,294** | **0,982** | **0,815** | **0,375** | **0,137** | **0,982** | **0,815** | **0,375** | **0,137** |
|  | |  | Obs. | 0,682 | 0,533 | 0,344 | 0,229 | 0,818 | 0,533 | 0,222 | 0,095 | 0,818 | 0,533 | 0,222 | 0,095 |
| **100** | | **1** | **Theo.** | **0,994** | **0,941** | **0,604** | **0,228** | **0,994** | **0,941** | **0,603** | **0,225** | **0,994** | **0,941** | **0,603** | **0,225** |
|  | |  | Obs. | 0,880 | 0,703 | 0,356 | 0,141 | 0,880 | 0,703 | 0,355 | 0,139 | 0,880 | 0,703 | 0,355 | 0,139 |
| **100** | | **1.5** | **Theo.** | **0,998** | **0,971** | **0,651** | **0,188** | **0,996** | **0,971** | **0,739** | **0,307** | **0,996** | **0,971** | **0,739** | **0,307** |
|  | |  | Obs. | 0,935 | 0,780 | 0,360 | 0,098 | 0,899 | 0,779 | 0,456 | 0,180 | 0,899 | 0,779 | 0,456 | 0,180 |
| **100** | | **2** | **Theo.** | **0,999** | **0,982** | **0,683** | **0,162** | **0,997** | **0,982** | **0,820** | **0,382** | **0,997** | **0,982** | **0,820** | **0,382** |
|  | |  | Obs. | 0,956 | 0,820 | 0,361 | 0,072 | 0,908 | 0,819 | 0,532 | 0,220 | 0,908 | 0,819 | 0,532 | 0,220 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table A4.8b  *Comparison between observed and expected power, when nominal alpha risk = 5%, three groups are compared and samples are extracted from double exponential distributions, where standard deviation is ..* | | | | | | | | | | | | | | | |
|  | |  |  | **Test** | | | | | | | | | | | |
|  | |  |  | ***F*-test** | | | | ***W*-test** | | | | ***F\**-test** | | | |
| **n1** | **n-ratio** | | **SDR :** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** | **0.5** | **1** | **2** | **4** |
| **20** | | **0.5** | **Theo.** | **0,192** | **0,214** | **0,220** | **0,232** | **0,463** | **0,209** | **0,095** | **0,062** | **0,282** | **0,209** | **0,124** | **0,086** |
|  | |  | Obs. | 0,101 | 0,132 | 0,173 | 0,216 | 0,278 | 0,140 | 0,073 | 0,051 | 0,156 | 0,129 | 0,092 | 0,072 |
| **20** | | **1** | **Theo.** | **0,452** | **0,338** | **0,196** | **0,124** | **0,593** | **0,329** | **0,136** | **0,073** | **0,443** | **0,336** | **0,188** | **0,111** |
|  | |  | Obs. | 0,245 | 0,194 | 0,134 | 0,102 | 0,354 | 0,196 | 0,095 | 0,059 | 0,238 | 0,191 | 0,128 | 0,091 |
| **20** | | **1.5** | **Theo.** | **0,627** | **0,424** | **0,181** | **0,076** | **0,647** | **0,413** | **0,176** | **0,083** | **0,532** | **0,421** | **0,245** | **0,132** |
|  | |  | Obs. | 0,373 | 0,239 | 0,110 | 0,056 | 0,393 | 0,237 | 0,114 | 0,064 | 0,290 | 0,236 | 0,155 | 0,102 |
| **20** | | **2** | **Theo.** | **0,732** | **0,487** | **0,169** | **0,050** | **0,677** | **0,472** | **0,213** | **0,093** | **0,586** | **0,481** | **0,292** | **0,151** |
|  | |  | Obs. | 0,471 | 0,274 | 0,093 | 0,033 | 0,418 | 0,271 | 0,130 | 0,070 | 0,326 | 0,270 | 0,178 | 0,112 |
| **30** | | **0.5** | **Theo.** | **0,320** | **0,310** | **0,267** | **0,242** | **0,657** | **0,303** | **0,118** | **0,068** | **0,440** | **0,305** | **0,162** | **0,099** |
|  | |  | Obs. | 0,150 | 0,178 | 0,200 | 0,222 | 0,397 | 0,188 | 0,087 | 0,056 | 0,227 | 0,178 | 0,115 | 0,083 |
| **30** | | **1** | **Theo.** | **0,659** | **0,490** | **0,267** | **0,143** | **0,793** | **0,482** | **0,187** | **0,085** | **0,654** | **0,489** | **0,261** | **0,133** |
|  | |  | Obs. | 0,360 | 0,274 | 0,170 | 0,113 | 0,503 | 0,276 | 0,120 | 0,066 | 0,354 | 0,272 | 0,165 | 0,105 |
| **30** | | **1.5** | **Theo.** | **0,817** | **0,602** | **0,263** | **0,098** | **0,841** | **0,594** | **0,251** | **0,102** | **0,748** | **0,600** | **0,341** | **0,165** |
|  | |  | Obs. | 0,520 | 0,342 | 0,150 | 0,067 | 0,555 | 0,343 | 0,151 | 0,075 | 0,431 | 0,340 | 0,206 | 0,121 |
| **30** | | **2** | **Theo.** | **0,889** | **0,675** | **0,260** | **0,070** | **0,864** | **0,666** | **0,309** | **0,119** | **0,797** | **0,671** | **0,410** | **0,194** |
|  | |  | Obs. | 0,624 | 0,392 | 0,134 | 0,042 | 0,584 | 0,392 | 0,178 | 0,084 | 0,478 | 0,389 | 0,240 | 0,135 |
| **40** | | **0.5** | **Theo.** | **0,463** | **0,404** | **0,315** | **0,256** | **0,796** | **0,395** | **0,145** | **0,074** | **0,591** | **0,400** | **0,202** | **0,112** |
|  | |  | Obs. | 0,207 | 0,226 | 0,226 | 0,229 | 0,506 | 0,236 | 0,101 | 0,061 | 0,304 | 0,226 | 0,137 | 0,093 |
| **40** | | **1** | **Theo.** | **0,808** | **0,621** | **0,334** | **0,164** | **0,904** | **0,615** | **0,239** | **0,097** | **0,805** | **0,621** | **0,330** | **0,156** |
|  | |  | Obs. | 0,475 | 0,353 | 0,204 | 0,124 | 0,631 | 0,356 | 0,145 | 0,073 | 0,470 | 0,352 | 0,200 | 0,117 |
| **40** | | **1.5** | **Theo.** | **0,919** | **0,740** | **0,346** | **0,119** | **0,935** | **0,734** | **0,326** | **0,121** | **0,878** | **0,739** | **0,432** | **0,198** |
|  | |  | Obs. | 0,645 | 0,440 | 0,190 | 0,077 | 0,685 | 0,441 | 0,188 | 0,085 | 0,561 | 0,439 | 0,256 | 0,138 |
| **40** | | **2** | **Theo.** | **0,958** | **0,808** | **0,353** | **0,091** | **0,949** | **0,802** | **0,404** | **0,145** | **0,912** | **0,805** | **0,516** | **0,236** |
|  | |  | Obs. | 0,744 | 0,502 | 0,178 | 0,052 | 0,715 | 0,503 | 0,226 | 0,097 | 0,614 | 0,500 | 0,301 | 0,157 |
| **50** | | **0.5** | **Theo.** | **0,595** | **0,492** | **0,358** | **0,270** | **0,884** | **0,484** | **0,171** | **0,080** | **0,716** | **0,489** | **0,240** | **0,124** |
|  | |  | Obs. | 0,271 | 0,272 | 0,250 | 0,235 | 0,602 | 0,283 | 0,114 | 0,064 | 0,381 | 0,274 | 0,156 | 0,099 |
| **50** | | **1** | **Theo.** | **0,900** | **0,727** | **0,400** | **0,186** | **0,959** | **0,722** | **0,293** | **0,110** | **0,899** | **0,726** | **0,396** | **0,179** |
|  | |  | Obs. | 0,580 | 0,429 | 0,240 | 0,134 | 0,733 | 0,432 | 0,171 | 0,079 | 0,577 | 0,429 | 0,237 | 0,128 |
| **50** | | **1.5** | **Theo.** | **0,967** | **0,836** | **0,426** | **0,142** | **0,976** | **0,832** | **0,401** | **0,141** | **0,946** | **0,835** | **0,517** | **0,229** |
|  | |  | Obs. | 0,745 | 0,531 | 0,231 | 0,089 | 0,784 | 0,532 | 0,225 | 0,095 | 0,671 | 0,530 | 0,304 | 0,155 |
| **50** | | **2** | **Theo.** | **0,986** | **0,891** | **0,443** | **0,112** | **0,982** | **0,888** | **0,493** | **0,171** | **0,965** | **0,890** | **0,608** | **0,276** |
|  | |  | Obs. | 0,832 | 0,599 | 0,223 | 0,061 | 0,811 | 0,600 | 0,274 | 0,110 | 0,725 | 0,597 | 0,361 | 0,179 |
| **100** | | **0.5** | **Theo.** | **0,948** | **0,810** | **0,550** | **0,341** | **0,996** | **0,804** | **0,310** | **0,111** | **0,975** | **0,809** | **0,421** | **0,182** |
|  | |  | Obs. | 0,609 | 0,503 | 0,365 | 0,272 | 0,891 | 0,507 | 0,182 | 0,081 | 0,726 | 0,503 | 0,254 | 0,131 |
| **100** | | **1** | **Theo.** | **0,998** | **0,962** | **0,900** | **0,291** | **1,000** | **0,961** | **0,600** | **0,178** | **0,998** | **0,962** | **0,900** | **0,287** |
|  | |  | Obs. | 0,903 | 0,733 | 0,407 | 0,188 | 0,961 | 0,733 | 0,303 | 0,112 | 0,902 | 0,733 | 0,405 | 0,184 |
| **100** | | **1.5** | **Theo.** | **1,000** | **0,989** | **0,736** | **0,262** | **1,000** | **0,989** | **0,705** | **0,247** | **1,000** | **0,989** | **0,805** | **0,381** |
|  | |  | Obs. | 0,967 | 0,839 | 0,433 | 0,145 | 0,977 | 0,839 | 0,412 | 0,144 | 0,948 | 0,839 | 0,523 | 0,235 |
| **100** | | **2** | **Theo.** | **1,000** | **0,996** | **0,783** | **0,238** | **1,000** | **0,996** | **0,811** | **0,312** | **1,000** | **0,996** | **0,884** | **0,464** |
|  | |  | Obs. | 0,985 | 0,893 | 0,451 | 0,115 | 0,983 | 0,893 | 0,503 | 0,176 | 0,966 | 0,892 | 0,613 | 0,281 |

), and will almost always detect highly skewed distributions, even when sample sizes are very small. However, there are still potential limitations. First, when sample sizes are small (i.e. *n* < 50; see Supplemental Material 1) all tests have low power to detect any other departures from the normality assumption. This is problematic since the normality assumption is especially crucial for small sample sizes (Supplemental Material 3 and 4). Second, with more than 50 subjects per group, the Shapiro-Wilk test will reliably detect departures from the normal distribution, even when those departures have no negative consequences for error rates or power[[6]](#endnote-5). Considering the limitations of tests for normality, it is often advised to combine the Shapiro-Wilk test with graphical methods (Ghasemi & Zahediasl, 2012; Öztuna, Elhan, & Tüccar, 2006).

In the same line of thinking, testing the homoscedasticity through a statistical test (such as Levene’s test) will often fail to reject the null hypothesis (i.e. lack of power) as soon as variances between groups differ slightly. This is problematic given that even small differences can invalid the *F*-test and Student’s *t*-tests (Delacre et al., 2017). In the next section, we will argue that the normality and equal variances assumptions are often unrealistic.

## **Is the Normality Assumption Realistic?**

It has been argued that there are many fields in psychology where the assumption of normality does not hold (Cain, Zhang, & Yuan, 2016). For example, Micceri (1989) reviewed 440 large datasets (i.e. n ≥ 400) published in a wide variety of journals between 1982 and 1984[[7]](#endnote-6), which contained psychometric and/or abilities measures. He found that the skewness and kurtosis of observed distributions seemed to be closer to an exponential curve than to a normal distribution (Figure 1).

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|  |
| *Figure 1*. Simulated standard exponential curve. |

It is now well known that in social and behavioral science, data are commonly heavy-tailed (Yuan, Bentler, & Chan, 2004). According to Wilcox (2005), for example, it commonly happens that distributions are very similar to a normal curve but with “thicker” tails than a normal distribution (Figure 2).

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|  |
| *Figure 2*. Mixed normal distribution where P(X~N(0,1))= .9 and where P(X~N(0,10))= .1, vs. N(0,1) and N(0,10). The distribution is very close to the N(0,1), but with higher kurtosis (25.3 vs. 3). |

There are also many situations where high kurtosis is associated with skewness. For example, when assessing a wellness score for the general population, data may be sampled from an asymmetric distribution with negative skewness, because most people are probably not depressed (Figure 3, left). An example is provided by the study of Heun, Burkart, Maier and Bech (1999), who evaluated the validity of the WHO Well-Being Scale (WBS) in the elderly, and found that the three versions of the WBS yielded highly skewed data. Moreover, when studying reaction times, data are often sampled from asymmetric distributions with positive skewness because it is uncommon to use much longer response time (Cain et al., 2016; Palmer, Horowitz, Torralba, & Wolfe, 2011; Van Zandt, 2000).

|  |
| --- |
|  |
| *Figure 3*. Example of fictive distributions, where skewness is negative (left) or positive (right) |

As these examples show, there are situations in psychological research where the normality assumption is implausible.

## **Is the Homogeneity of Variance Assumption Unrealistic ?**

The second important assumption in many parametric tests is that variances are homogeneous. Discussions in the literature have pointed out that this assumption is problematic in psychological research (Erceg-Hurn & Mirosevich, 2008; Grissom, 2000). In a previous paper (Delacre et al., 2017), we have discussed three different causes of unequal standard deviations across groups of observations: the variability inherent to the use of measured variables, the variability induced by quasi-experimental treatments on measured variables, and the variability induced by different experimental treatments on randomly assigned subjects.

First, psychologists often use measured variables (e.g. age, gender, educational level, ethnic origin, depression level, etc.) instead of random assignment to conditions. Now, prior to any treatment, parameters of pre-existing groups can vary largely from one population to another, as suggested by Henrich, Heine and Norenzayan (2010). This can be observed when one compares different countries on cultural dimensions. For example, Green, Deschamps and Paez (2005)have shown that the scores of competitiveness, self*-*reliance and interdependence are more variable in some groups than in others[[8]](#endnote-7). Many other examples could be cited where constructs have different variances when pre-existing groups from different cultures, religions, or ethnicity are compared (see for example Adams, Van de Vijver, De Bruin, & Bueno Torres, 2014; Beilmann, Mayer, Kasearu, & Realo, 2014; Church et al., 2012; Cohen & Hill, 2007; Haar, Russo, Suñe, & Ollier-Malaterre, 2014; Montoya & Briggs, 2013). Differences in variability between groups are also often plausible in other fields, such as when examining gender differences or in educational psychology, for example when different school systems are compared (Delacre et al., 2017). In this last example, groups are defined in order to have different variability: As soon as a selective school admits its students based on the results of aptitude tests, the variability will be smaller compared to a school that accepts all students.

Second, a quasi-experimental treatment can have a different impact on variances between pre-existing groups. For example, in the field of linguistics and social psychology, Wasserman and Weseley (2009) investigated the impact of language gender structure on sexist attitudes of women and men. They tested differences between sexist attitude scores of subjects who read a text in English (i.e. a language without grammatical gender) or in Spanish (i.e. a language with grammatical gender). The results showed that for a reason that is undetermined by the authors, the women’s score on the sexism dimension was more variable when the text was read in Spanish than in English (SDspanish=.80 > SDenglish=.50). For men, the reverse was true (SDspanish=.97 < SDenglish=1.33; Wasserman & Weseley, 2009).

Third, even when variances of groups are the same before treatment (due to a complete randomization in the group assignment), unequal variances can emerge later, as a consequence of an experimental treatment (Bryk & Raudenbush, 1988; Cumming, 2013; Erceg-Hurn & Mirosevich, 2008; Keppel & Wickens, 2004). For example, Koeser & Sczesny (2014) have compared arguments advocating either masculine generic or gender-fair language with control messages in order to test the impact of these conditions on the use of gender-fair wording (measured as a frequency). They report that the standard deviations increase after treatment, in all experimental conditions.

# Simulations Comparing the *F*-test vs. *W*-test vs. *F\**-test

We performed simulations to examine the Type 1 error rate and power for different underlying distributions for the *F*-test, *W*-test and *F\**-test. The differences between the three tests are mathematically explained in the appendix, which mainly concern the way standard deviations are pooled across groups.

## **Type 1 Error Rate of the *F*-test vs. *W*-test vs. *F\**-test**

### **When the normality assumption is met.** To examine the differences in Type 1 error rate between *F*-test, *W*-test and *F\**-test, we simulated 1,000,000 studies under the null hypothesis (where there is no difference between the means in each group) for four scenarios. For each scenario, we examine the *p*-value distribution. When 5% of the *p*-values fall below 0.05, the Type 1 error rate is controlled as intended. Each scenario was repeated twice, once for an ANOVA with two groups, and once for an ANOVA with three groups (as explained in the appendix, when comparing two groups, *W*-test and *F\**-test are mathematically identical and should yield identical error rates). The Type 1 error rate of the three tests under all scenarios are summarized in Table 1. In scenario 1, the variances are the same in each group (SD-ratio = 1; homoscedasticity assumption met) and sample sizes are unequal (n=20 in the last group; n=40 in all other groups). Table 1 shows that the Type 1 error rate is controlled as intended for all three ANOVA tests, when comparing 2 and 3 groups. In Scenario 2, the variances differ between groups (SD-ratio = the ratio between the biggest standard deviation and the smallest standard deviation = 4) but sample sizes are equal (n= 40 in all groups). Table 1 shows that only *W*-test controls the Type 1 error rate as intended when comparing three groups. In Scenario 3, both sample sizes and variances were unequal between groups and the larger variance is associated with the larger sample size (SD-ratio =4; n=80 in the last group; n=40 in all other groups). Table 1 again shows the *W*-test controls better the Type 1 error rate than the *F*-test. Finally, Scenario 4 is the same as Scenario 3, but the larger variance is associated with the smaller sample size (SD-ratio = 4; n=20 in the last group; n=40 in all other groups, with the same results as Scenario 3).

|  |  |
| --- | --- |
| k = 2 | k = 3 |
|  |  |
| **Figure 4**: *P*-value distributions for the *F*-test, *W*-test and *F\**-test under the null hypothesis when variances are unequal between groups (SD-ratio =4) and sample sizes are equal between groups (n= 40 in all groups), as a function of the number of groups to compare. | |

As shown in Table 1, when there are only two groups to compare, and as long as the variances are equal between groups, the *p*-value distribution of the *F*-test is close of the uniform, as expected. When sample sizes are equal between groups, the impact of unequal variances is very low and the *p*-value distribution remains very close to a uniform distribution. However, when there is a positive (or negative) correlation between sample sizes and standard deviations (i.e. the larger variance is associated with the larger – or smaller - sample size), the frequency of *p*-values under 5 percent decreases (or increases).

When there are three groups to compare, the *p*-value distribution of the *F*-test is uniform only when variances are equal. When variances are unequal, the frequency of *p*-values under 5 percent (i.e. the Type 1 error rate) differs from the nominal 5%, even when sample sizes are equal between groups (as shown in Figure 4). In this latter case, the *F*-test becomes more liberal.

This tendency can be generalized: when the number of group increases, the test becomes increasingly liberal; the Type 1 error rate is too low when there is a positive correlation between sample sizes and standard deviations, but too high when there is either a negative correlation between sample sizes and standard deviations or heteroscedasticity with balanced designs[[9]](#endnote-8). The *F\**-test is robust against unequal variances when there are two groups to compare (Table 1). When there are three groups to compare, the test is less affected by violations of the assumption of equal variances than the *F*-test, but the Type 1 error rate still increases when there are unequal variances between groups. Additional simulations, presented in the Supplemental Material, show that the test gets more liberal as the sample size diminishes, and as the SD-ratio and the number of groups to compare increases. Finally, *W*-test yields a more stable Type 1 error rate, regardless the number of groups that is compared, and the SD-ratio.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1.  *Comparison of Type 1 error rate of the F-test, W-test and F\*-test, as a function of the number of groups* | | | | | | | |
|  | Two groups | | |  | Three groups | | |
| Scenario | F | F\* | W |  | F | F\* | W |
| 1 | 0.050 | 0.050 | 0.050 |  | 0.050 | 0.050 | 0.050 |
| 2 | 0.053 | 0.050 | 0.050 |  | 0.078 | 0.072 | 0.050 |
| 3 | 0.009 | 0.050 | 0.050 |  | 0.016 | 0.072 | 0.050 |
| 4 | 0.155 | 0.050 | 0.050 |  | 0.192 | 0.071 | 0.050 |
| *Note*. Type 1 error rates for the F-test, W-test and F\*-test are compared when variances are equal (SD-ratio=1) and sample sizes are unequal between groups (n=20 in the last group; n = 40 in all other groups; Scenario 1), when variances are unequal between groups (SD-ratio=4) and sample sizes are equal (n = 40 in all groups; Scenario 2), positively correlated with the variance (SD-ratio=4, n=80 in the last group, n = 40 in all other groups; Scenario 3), or negatively correlated with the variance (SD-ratio=4, n=20 in the last group; n=40 in all other groups; Scenario 4). | | | | | | | |

### **When the normality assumption is not met.** While the *W*-test is more robust than both the *F*-test and *F\**-test when there are unequal variances, it is less robust than the two other tests when the normality assumption is not met (Supplemental Material 3). The *W*-test is more affected by heavy-tailed and skewed distributions than the *F*-test, becoming more conservative with heavy-tailed distributions (Table A3.2 and A3.3), and more liberal with skewed distributions (Table A3.4, A3.5, A3.6 and A3.7). It happens that the *W*-test becomes even more liberal when highly skewed distributions are combined with unequal variances and sample sizes between groups.

We offer the following recommendations. When the data is not normally distributed, and variances are unequal, the *F*-test requires 20 subjects per group to control the Type 1 error rate within an interval of .025 to .075 (Bradley,1978). However, regardless of the sample size, the Type 1 error rate will commonly be out of this interval when variances are unequal (the same holds for the *F\**-test). When distributions look symmetric or are moderately skewed (see Supplemental Material 3) *W*-test can be used with only 20 subjects per group. With highly skewed distributions, at least 50 subjects per group are required (when comparing a maximum of four groups), and with even more groups, a larger sample size per group is required.

Nevertheless, the *W*-test is preferable to the *F*-test. Finally, highly skewed distributions are easier to detect with a Shapiro-Wilk test than unequal variances with a test of homogeneity of variances (Delacre et al., 2017).

## **Power for *F*-test vs. *W*-test vs. *F\**-test**

In addition to the Type 1 error rate, the Type 2 error rate is an important aspect of a test. In order to examine the power of the *F*-test, *W*-test and *F\**-test, we performed simulations in which we introduced a true effect (the mean = 1 in the last group, mean = 0 in all other groups).

We also manipulated the distribution and variances across groups. In order to manipulate normality independently of the equality of variances, we relied on skewness and/or kurtosis indicators (Supplemental Material 4).

First, it is often believed that the *W*-test and *F\**-test are less powerful than the *F*-test when the assumption of the *F*-test are met[[10]](#endnote-9). Yet when both assumptions are met, our simulations show that the loss of power is never above 1.5% when performing a *W*-test, and never above 0.7% when performing a *F\**-test. This loss is marginal in comparison with the deviation in the Type 1 error rate from the nominal 5% when performing a *F*-test with groups of unequal variances. Moreover, the differences in power between *F*-test and both *W*-test and *F\**-test tend towards zero when the number of subjects per group increases[[11]](#endnote-10). When data are extracted from skewed distributions, the loss of power can reach up to 6.4% when performing *W*-test or *F\**-test instead of the *F*-test. However, the difference between both statistics considerably decreases when sample sizes increase: with at least 50 subjects per group, the *F*-test has 2.1% higher power than the other two tests, and the bigger is the sample size, the smaller is the difference. Remember that with less than 50 subjects per group, *W*-test should be avoided when distributions are highly skewed (as detected by the Shapiro-Wilk test)[[12]](#endnote-11).

A last topic concerns a common confusion between kurtosis and standard deviation. In previous work examining the power of non-normal distributions, Wilcox (1998) concluded that there is a loss of power when performing a test comparing means when distributions are heavy-tailed (e.g. double exponential or some mixed normal distribution; Figure 2). This finding is based on the argument that heavy-tailed distributions are associated with bigger standard deviations than normal distributions, and that the effect size for such distributions is therefore smaller (Wilcox, 2011). However, while the standard scale parameter of the double exponential distribution is bigger than the standard scale parameter of the normal distribution (standard scale parameter of the normal distribution = β = ), DeCarlo (1997) explains that kurtosis and SD are totally independent, meaning that one can find distributions that have similar SD but different kurtosis. When heavy-tailed distributions have equal standard deviations and SD-ratios as normal distributions, there are no substantial differences in power as a function of the kurtosis of the underlying distribution (Supplemental Material 4).

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# Conducting Shapiro-Wilk test and *W*-test in R or SPSS

## **Conducting Shapiro-Wilk test**

In R, the Shapiro-Wilk test for each compared groups can be run by the function “shapiro.test”, using the following syntax: **shapiro.test(*data.name*$*dv.name[data.name$iv.name= =x]*)[[13]](#endnote-12),** where x corresponds to one level of the iv.

In SPSS, the Shapiro-Wilk test can be run using the following syntax:

**EXAMINE VARIABLES=DV BY IV**

**/PLOT NPPLOT**

Figure 5 shows the output, obtained in SPSS, when performing a Shapiro-Wilk test on data summarized in Table A1.

|  |
| --- |
|  |
| *Figure 5*. Output in SPSS |

## **Conducting *W*-test**

In R, the *W*-test can be run by the function “oneway.test”, using the following syntax: **oneway.test(*dv.name* ~ *iv.name*, data=*data.name*, var.equal=FALSE)** [[14]](#endnote-13).

The last argument is used to specify that the *W*-test should be used instead of the *F*-test, relying on the assumption of equal variances. This argument is optional, and when the var.equal is not specified, the *W*-test is reported by default.

In SPSS, the *W*-test can be run using the following syntax:

**ONEWAY *dv.name* BY *iv.name***

**/STATISTICS WELCH**

Figure 6 shows the output, obtained in SPSS, when performing a *W*-test on data summarized in Table A1. As one can see, the degrees of freedom in the numerator of *W*-test and *F*-test are the same. However, in the degrees of freedom in the denominator of *W*-test, there are decimals.

|  |
| --- |
|  |
| *Figure 6*. Output in SPSS |

# Conclusion and Recommendations

Four recommendations follow from our analysis and simulations:

1) By default use *W*-test instead of the *F*-test. The *F*-test and *F\**-test should be avoided, because the equal variances assumption is often unrealistic, tests of the equal variances assumption will often fail to detect differences when these are present, the loss of power is very small (and often even negligible), and the gain in Type 1 error control is considerable under a wide range of realistic conditions.

2) Use the Shapiro-Wilk test to detect departures from normality (combined with graphical methods). Contrary to the Kolmogorov-Smirnov test, the Shapiro-Wilk test will almost always detect distributions with high skewness, even with very small sample sizes. With small sample sizes, the Type 1 error rate will not be controlled when using the *W*-test, and detecting departures for normality is therefore especially important in small samples. When comparing at most four groups, the Shapiro-Wilk test should be used when there are less than 50 observations per groups. When the number of groups in the ANOVA is greater than four, the Shapiro-Wilk test should be used when there are less than 100 subjects per groups. When normality cannot be assumed because of high kurtosis or high skewness, we recommend the use of alternative tests that are not based on means comparison, such as the trimmed means test. For more information, see Erceg-Hurn and Mirosevich (2008).

3) Perform prior power-analysis. Fifty subjects per groups are generally enough in order to control the type 1 error. However, prior-analysis are important in order to determine the required sample sizes to achieve a sufficient power (.80 or .95, depending on the criterion). It will depend, among others, on the effect size and the sample sizes ratio.

4) Use balanced designs (i.e. same sample sizes between groups) when possible. When using the *W*-test, the Type 1 error rate is a function of criteria such as the skewness of the distributions, and whether skewness is combined with unequal variances and sample sizes between groups. Our simulations show that the Type 1 error rate control is in general slightly better for balanced designs.

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# Appendix

## **The Mathematical Differences Between the *F*-test vs. *W*-test vs. *F*\*-test**

In this section, we will explain the mathematical differences in how the *F*-test, *W*-test and *F\**-test are computed, with a focus on the differences in how SD are pooled across groups.

As shown in formula 1, The *F* statistic is calculated by dividing the inter-group variance by a pooled error term, where and nj are respectively the variance estimates and the sample sizes from each independent group, and where *k* is the number of independent groups:

|  |  |  |
| --- | --- | --- |
|  | F = | (1) |

The degrees of freedom in the numerator (formula 2) and in the denominator (formula 3) of the *F*-test are computed as follows:

|  |  |  |
| --- | --- | --- |
|  | Dfn = k-1 | (2) |
|  | Dfd = N-k, where N = | (3) |

As a generalization of the Student’s *t*-test, the *F*-test is calculated based on a pooled error term, which implies that all samples are estimates of a common population variance. The *F*-test suffers from the same limitations as the *t*-test when sample sizes are unequal between groups, in that the Type 1 error rate is no longer controlled at the desired level when variances are unequal between groups. When the larger variance is associated with the larger sample size, there is a decrease in the Type 1 error rate (Nimon, 2012; Overall, Atlas, & Gibson, 1995), because the error term increases, and therefore, the *F-*value decreases, leading to fewer significant findings than expected with a specific type 1 error level. When the larger variance is associated with the smaller sample size, the Type 1 error rate is inflated (Nimon, 2012; Overall et al., 1995). This inflation is caused by the under evaluation of the error term, which increases the *F-*value, and thus leads to more significant results than expected based on the nominal Type 1 error level. Moreover, when the number of groups increases, the *F-*test becomes increasingly liberal as soon as the variances of the distributions in each group are not similar, even when sample sizes are equal between groups.

To address the problems with error control in the *F*-test when variances are unequal, several authors have proposed alternative approaches to statistical tests on more than two means, which do not rely on the homogeneity of variances assumption (e.g., Welch, 1951). Tomarken and Serlin (1986) have shown that from the available alternatives, *F\**-test and *W*-test are the best choice. Both tests are available in SPSS, which is a widely used software in psychological science (Hoekstra et al., 2012). The *F\** statistic proposed by Brown and Forsythe (1974) is computed as follow:

|  |  |  |
| --- | --- | --- |
|  | *F\** = | (4) |

Where xj and are respectively the group mean and the group variance, and is the overall mean.

As can be seen in formula 4 the numerator of the *F\** statistic is equal to the sum of squares between groups (which is equal to the numerator of the *F* statistic when one compares two groups). In the denominator of the statistic, the variance of each group is weighted by 1 minus the relative frequency of each group, so that the variance associated with the group with the smallest sample size is given more weight. As a result, when the larger variance is associated with the larger sample size, *F\** is larger than *F*, because the denominator decreases, leading to more significant findings compared with the *F*-test. On the other hand, when the larger variance is associated with the smaller sample size, *F\** is smaller than *F*, because the denominator increases, leading to fewer significant findings than expected with the *F*-test. The degrees of freedom in the numerator and in the denominator of *F\**-test are computed as follow:

|  |  |  |
| --- | --- | --- |
|  | Dfn = k-1 | (5) |
|  | Dfd= | (6) |

As shown in our simulations, the *F\**-test appears to be more robust than the *F*-test in many situations where there are unequal variances between groups, when looking at the Type 1 error rate, but in many circumstances, it is too liberal. Our simulations also show that the *W*-test has better Type 1 error control than both *F*-test and *F\**-test when there are unequal variances between groups. As can be seen in formula 7, the squared deviation between groups means and the general mean are weighted by instead of nj in the numerator of the *W*-test (Brown & Forsythe, 1974).

|  |  |  |  |
| --- | --- | --- | --- |
|  | *W* = , | | (7) |
| where | , |

The degrees of freedom of *W*-test are approximated as follows:

|  |  |  |
| --- | --- | --- |
|  | Dfn = | (8) |
|  | Dfd = | (9) |

When there are only two groups to compare, the *F\**-test and *W*-test test are identical (i.e., they have exactly the same statistical value, degrees of freedom and significance). However, when there are more than two groups to compare, the tests differ.

To better understand how to compute all statistics, a set of fictional raw data simulate the example of a three-groups design. A summary is presented in Table A1. The complete example is available on Github. The DV is a score that can vary from 0 to 40. The IV is a three-level factor A (levels = A1, A2 and A3).

|  |  |  |  |
| --- | --- | --- | --- |
|  | A1 | A2 | A3 |
| ni | 41.00 | 21.00 | 31.00 |
|  | 24 | 23 | 27 |
|  | 81.75 | 10.075 | 38.40 |

Table A1. *Summary of the data of the fictive case*

The global mean (i.e. the mean of the global dataset) is a weighted mean of the group means:

The *F*-test statistic and degrees of freedom are computed by applying formulas 1, 2 and 3:

F = 2.377

dfn = 3-1 = 2

dfd = 93-3 = 90

The *F\**-test and his degrees of freedom are computed by applying formulas 4, 5 and 6.

*F\** = 3.088

|  |  |
| --- | --- |
| dfn = 3-1 = 2 | |
| dfd = 81,149 | |
| where | 79,11 |

Finally, the *W*-test and his degrees of freedom are computed in applying formulas 7, 8 and 9:

|  |  |
| --- | --- |
| W = 4.606 | |
| Where | w = 3,39  24,10 |
| dfn = 3-1 = 2 | | |
| dfd = = 59,32 | | |

One should notice that in this example, the biggest sample size has the biggest variance. As previously mentioned, it means that the *F*-test will be too conservative, because the *F* value decreases. The *F*\* ANOVA will also be a little too conservative, even if the test is less affected than F-test. As a consequence: *W* > *F\** > *F*.

1. The null hypothesis of nonparametric tests, such as the Mann-Whitney *U* test and Kruskal-Wallis, assumes that the distributions are the same between groups. Any departure to this assumption, such as unequal variances, will therefore lead to the rejection of the assumption of equal distributions (Grissom, 2000; Nachar, 2008; Neuhäuser & Ruxton, 2009; Tomarken & Serlin, 1986; Zimmerman, 2000). Other alternatives, such as trimmed means test, exist (Wilcox et al., 2013). The null hypothesis of the trimmed means test assumes that trimmed means are the same between groups. A trimmed means is a mean computed on data after removing the lowest and highest values of the distribution (Erceg-Hurn & Mirosevich, 2008). Trimmed means and means are equal when data are symmetric. On the other side, when data are asymmetric, trimmed means and means differ. [↑](#endnote-ref-1)
2. We do not include the Alexander-Govern and James’ tests (i.e. two alternatives that are robust against unequal variances between groups) because these tests are not available in all statistical software (such as SPSS). However, a pre-test revealed us that these tests give results very close to the *W*-test meaning that they have very similar strengths and limitations. When data are symmetrically distributed, the biggest difference we found between the *W*-test and respectively Alexander-Govern test and James test is .008. When data are skewed, with unequal skewness between groups, it can increase to maximum .01. [↑](#endnote-ref-2)
3. In the whole paper, everytime we mention « unequal variances », we refer to population variances data are extracted from, and not sample variances. In our simulations, we always manipulated the population variances and we don’t need to infer it from the sample. [↑](#endnote-ref-3)
4. The Shapiro-Wilk test is based on the correlation between the observed data and their corresponding normal score (i.e. the vector or quantile of the observed data, and the vector of quantile that should be obtained if data were normally distributed; Ghasemi & Zahediasl, 2012; Öztuna, Elhan, & Tüccar, 2006). [↑](#endnote-ref-4)
5. Note that it is not possible to compare results when k=2 and when k=3, because the sample size is not the same. [↑](#footnote-ref-1)
6. For example, in a previous paper, we have shown that when comparing two groups when data are uniformly distributed (i.e. kurtosis = 1.8) between groups, tests comparing means are still valid, both in terms of the Type 1 error rate and the power (Delacre, Lakens, & Leys, 2017). [↑](#endnote-ref-5)
7. *Applied Psychology*, *Journal of Research in Personality*, *Journal of Personality*, *Journal of Personality Assessment*, *Multivariate Behavioral Research*, *Perceptual and Motor Skills*, *Applied Psychological Measurement*, *Journal of Experimental Education*, *Journal of Educational Psychology*, *Journal of Educational Research*, and *Personnel Psychology*. [↑](#endnote-ref-6)
8. Among others, the score of competitiveness in Switzerland (*M* = 1.9; *SD* = .57) and Spain (*M* = 1.65; *SD* = .56) are less variable than the score of competitiveness in Italy (*M* = 2.12; *SD* = .79) or France (*M* = 2.28; *SD* = .75). [↑](#endnote-ref-7)
9. To yield a robust test, the Type 1 error rate has to be sufficiently close to the nominal 5%. In order to assess the robustness of the three tests in our simulations, according to Bradley (1978) we consider the Type 1 error rate is ‘close enough’ to the nominal 5% if it’s value falls in the interval [0.025; 0.075]. [↑](#endnote-ref-8)
10. We will only compare results of the *W*-test and the *F*-test when the assumption of equality of variances is met because when variances between groups are unequal, results of the *F*-test are not valid. When there is a negative correlation between sample sizes and SD or when there are unequal SD with the same sample sizes between groups, the power of the *F*-test is overestimated (Power*F*-test > Power*W*-test); when there is a positive correlation between sample sizes and SD, the power of the *F*-test is underestimated (Power*F*-test < Power*W*-test). [↑](#endnote-ref-9)
11. For example, with at least 50 subjecst per groups, the difference between the *p*-value of the *F*-test and *W*-test is 0.5%. The difference between the *p*-values of the *F*-test and *F\**-test is 0.1%. [↑](#endnote-ref-10)
12. Note that 50 subjects per group is enough in order to achieve robustness in terms of Type 1 error rate, however, it is also important to have a good power. The required number of subjects in order to achieve a sufficiant power will be a function of parameters such as the effect size, the sample sizes ratio, and the power criterion. In general, the power is acceptable at .80 (Mackinnon, 2013) and should ideally achieve .95. [↑](#endnote-ref-11)
13. Details can be obtained in typing « ?shapiro.test” in the R console, and a practical example is given in the following link: … [↑](#endnote-ref-12)
14. Details can be obtained in typing "?oneway.test" in the R console, and a practical example is given in the following link: … [↑](#endnote-ref-13)